VOLUME 24 OCTOBER, 1936 NUMBER 10

PROCEEDINGS
of
The Institute of Radio Engineers

Institute of Radio Engineers Forthcoming Meetings

ROCHESTER FALL MEETING
November 16, 17, and 18, 1936

CHICAGO SECTION
October 23, 1936

CINCINNATI SECTION
October 20, 1936

CLEVELAND SECTION
October 22, 1936

DETROIT SECTION
October 16, 1936

EMPORIUM SECTION
October 15, 1936

LOS ANGELES SECTION
October 20 1936

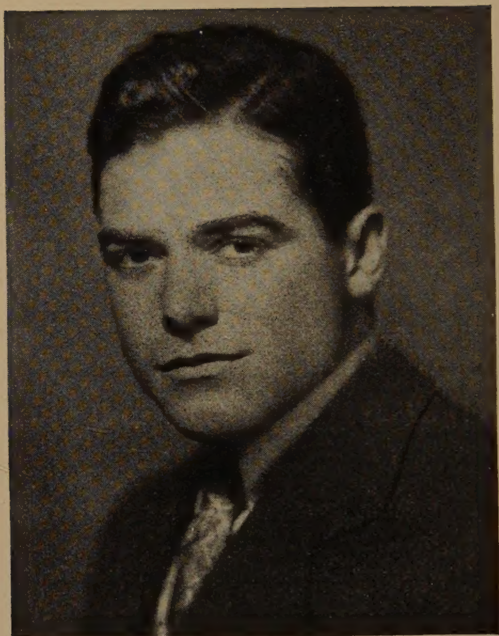
NEW ORLEANS SECTION
October 7, 1936

NEW YORK MEETING
October 7, 1936

PHILADELPHIA SECTION
October 8, 1936

PITTSBURGH SECTION
November 20, 1936

WASHINGTON SECTION
October 12, 1936



B. J. THOMPSON

Recipient of the Morris Liebmann Memorial Prize, 1936

Browder J. Thompson was born in Roanoke, Louisiana, on August 14, 1903. He attended the University of Washington and received a Bachelor of Science degree in electrical engineering in 1925.

From 1926 to 1931 he was a member of the research laboratory of the General Electric Company at Schenectady, New York, where he was active in the design and development of tubes both for radio and industrial purposes. In 1931 he joined the research and development laboratory of the RCA Radiotron Company at Harrison, New Jersey, where he was placed in charge of electrical research. His work in that organization on vacuum tubes of small physical dimensions for ultra-high-frequency uses resulted in the presentation to him of the Morris Liebmann Memorial Prize for 1936 with the citation "for his contribution to the vacuum tube art in the field of very high frequencies."

Mr. Thompson is a member of the American Physical Society. He joined the Institute as an Associate member in 1929, transferring to Member in 1932. He has been active in standardization work of the Institute for several years.

INSTITUTE NEWS AND RADIO NOTES

Committee Work

Broadcast Committee

Three meetings of the Broadcast Committee were held to consider the desirability of the Institute's preparing a statement for presentation at hearings of the Federal Communications Commission starting on October 5, 1936, on practically the entire subject of radio broadcasting. A preliminary review of the field was made and those subjects on which it was felt the Institute might make worth-while contributions were discussed in detail.

The meeting held on August 6 was attended by E. L. Nelson, chairman; H. A. Chinn, (representing E. K. Cohan), G. D. Gillette, Alan Hazeltine (representing H. A. Wheeler), J. V. L. Hogan, C. W. Horn, C. B. Jolliffe, and H. P. Westman, secretary.

The August 17 meeting was attended by E. L. Nelson, chairman; G. D. Gillette, R. N. Harmon, Alan Hazeltine (representing H. A. Wheeler), J. V. L. Hogan, C. B. Jolliffe, W. B. Lodge (representing E. K. Cohan), V. E. Trouant, and H. P. Westman, secretary.

The September 1 meeting was attended by E. L. Nelson, chairman; G. D. Gillette, R. N. Harmon, J. V. L. Hogan, C. W. Horn, C. M. Jansky, Jr., J. E. Young, and H. P. Westman, secretary.

Technical Committee on Electroacoustics

The Technical Committee on Electroacoustics met on July 10 in the Institute office and those present were H. F. Olson, chairman; Sydney Blumenthal, C. H. G. Gray, Benjamin Olney, V. E. Whitman, and Julius Weinberger.

The committee reviewed some preliminary material on loud-speaker testing which, after revision, was considered satisfactory as a final report to go to the Institute's Standards Committee.

Institute Meetings

Cleveland Section

R. M. Pierce, chairman, presided at the April 23 meeting of the Cleveland Section held at Case School of Applied Science and at which thirty-eight members and guests were present.

A paper on "Design and Construction of Indicating Instruments" was presented by H. L. Olesen of the Weston Electrical Instrument

Corporation. The speaker covered the construction of indicating instruments, illustrating his remarks with parts used in them. He stated that pointers are thin wall aluminum tubing, wires as small as a thousandth of an inch in diameter are enamel insulated, springs are alloyed especially for meter service, pivots are finer than needle points, and jewels are of sapphire. Various types of instruments were described and their advantages and disadvantages pointed out. The dynamometer type was indicated as being comparatively insensitive and the repulsion type as inefficient. In both of these types the field is supplied from the circuit under measurement. The use of a permanent magnet for the field as in the d'Arsonval type of movement results in a much greater ruggedness and sensitivity. Copper-oxide rectifiers were discussed and their limitations pointed out. Thermal meters with iron-constantan or platinum—platinum-iridium junctions are used for frequencies below a hundred kilocycles. Above that frequency, a correction factor must be used and care employed in shielding the meter circuit from stray pickup. The paper was concluded with an exhibition of allied equipment.

Connecticut Valley Section

The Connecticut Valley Section met on April 30 at the Hotel Charles in Springfield, Mass. There were forty present and M. E. Bond, chairman, presided.

Two papers were presented, the first of these was by F. H. Scheer of the F. W. Sickles Company on "Diode Coupling Transformers." He discussed chiefly the effect of the ratio of the alternating-current to direct-current impedance in a conventional diode circuit with two diodes tied together and the automatic volume control system connected to one end of the volume control. In the discussion which followed, the possibility of improving the ratio by coupling the automatic volume control diode to the audio-frequency diode through a small capacitance which would present a relatively high impedance to audio frequencies was considered as was the best design for the intermediate-frequency transformer which couples the final intermediate-frequency amplifier to the diode circuit.

The second paper was on the "Manufacture of Fibre and Phenolic Products" by C. F. Cary and Mr. Allison of the Spaulding Fibre Company. The major portion of the paper was in the form of motion pictures which covered the details of manufacture. The discussion which followed the paper considered not only the manufacturing process but the application of products.

The May meeting of the Connecticut Valley Section was held on

the 28th at the Dunham Laboratory of Electrical Engineering in New Haven, Conn. Chairman Bond presided and the attendance was fifty-five, twenty of whom were present at the informal dinner which preceded the meeting.

A paper on "Accurate Methods of Measuring both Inductance and Capacitance" was presented by R. F. Field of the General Radio Company. It was pointed out that it is necessary to measure resistance values from 10^{-6} to 10^{12} ohms, inductances from 10^{-8} to 10^{3} henrys, and capacitances from 10^{-14} to 10^{-3} farads. Existing technique permits extremely accurate measurements of resistance from 10 to 10^{-4} ohms. Inductance from 10^{-3} to 10 henrys, and capacitance from 10^{-9} to 10^{-6} farads.

Resistance cannot be calculated accurately because of variations in materials. A simple Wheatstone bridge has errors due to thermoelectric effect, contact resistance, and the resistance of connecting links. The greatest accuracy may be obtained by substituting unknowns which are made exactly equal to the standard resistor. Capacitance measurements may be in error because of stray capacitances, connectors, and temperature. Inductance measurements are considered most difficult because of inductors having resistance and stray capacitance which change the natural period of the coil. There is no precision variable inductor available. It was pointed out that resistance, inductance, and capacitance standards have been compared to within an accuracy of 0.01 per cent.

Los Angeles Section

A meeting of the Los Angeles Section was held on June 16 at the Los Angeles Junior College under the chairmanship of C. R. Daily. Seventy-five were present and there were twenty-eight at the dinner which preceded the meeting.

A paper on "The Feed-Back Amplifier as Applied to Radio" was presented by F. E. Terman of Stanford University. It was pointed out that the feed-back amplifier circuit makes possible amplifiers having reduced distortion, reduced noise, hum, etc., more stable amplification, reduced phase shift, and any desired ferquency response. These results are obtained by feeding back to the input of the amplifier a portion of the output which is large in comparison to the input and reversed in phase, through a suitably designed network. If the product of gain times feedback be such that its locus does not enclose the point (+1) the amplifier will not oscillate. It was pointed out that improvement in performance was made at the expense of reduced gain in the amplifier. The paper was discussed by Messrs. Daily, Downs, and Kiernan.

San Francisco Section

The July 17 meeting of the San Francisco Section was held at the Bellevue Hotel with V. J. Freiermuth, vice chairman, presiding. There were forty-eight present at the meeting and eighteen attended the informal dinner which preceded it.

H. R. Lubcke, director of television of the Don Lee Broadcasting Company, presented a paper on "Five Years of Television Broadcasting." He described first existing conditions in the television field both from the regulatory as well as engineering and operating aspects. He then outlined the history and activities of his own organization for the past five years. Mimeographed data to permit the construction of receiving equipment to pick up the Don Lee television broadcasts were distributed. At the present time they are broadcasting high fidelity films daily except Sunday and have been in operation almost every day for the past four years.

Personal Mention

R. M. Arnold formerly with Philco Radio and Television Corporation has established the Arnold Engineering Company in Chicago, Ill.

L. B. Bender, Lieutenant Colonel, U.S.A., has been made Chief of the Research and Development Division in the Office of the Chief Signal Officer, Washington, D. C., having formerly been stationed at Fairfield, Ohio.

R. M. Beusman has become manager of the western division of the Radio Condenser Company of Chicago, Ill., having previously been with the Reliance Die and Stamping Company.

T. R. W. Bushby has left Raycophone, Ltd., to join the engineering staff of Amalgamated Wireless (Australasia), Ltd., at Ashfield, New South Wales, Australia.

J. R. Pattee previously with Fairchild Aerial Camera Corporation has established the Pattee Scientific Development Company at Long Island City.

D. H. Vance has left the RCA Victor Company to become affiliated with the Electronic Equipment Corporation of Philadelphia, Pa.

E. D. Whitehead has joined the staff of Pye Radio Ltd., Cambridge, England, having previously been with H. Clarke and Company, Ltd.

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TECHNICAL PAPERS

THE DESIGN OF DOUBLET ANTENNA SYSTEMS*

By
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Summary—*An antenna system has been designed for all-wave radio reception at 0.54 to eighteen megacycles with reduction of noise from causes near the receiver. The system includes doublet antenna, transmission line and coupling filters. The fifteen-meter double V doublet is designed on the basis of impedance curves, for satisfactory balanced operation at six to eighteen megacycles. Unbalanced operation is employed at the lower frequencies. Each of the antenna-to-line and line-to-receiver filters includes low band and high band transformers together passing the entire frequency range with little attenuation. The antenna transformers are included in contiguous band filters for switching the antenna between unbalanced and balanced operation. The receiver transformers are included in a continuous band filter. Balanced operation of the line is carefully preserved. The over-all loss is measured with the aid of a special dummy antenna. The over-all loss, minus the line attenuation, is between one and five decibels relative to the maximum power theoretically obtainable by tuning the antenna to each signal. The receiver is designed to operate uniformly as a 400-ohm load at any frequency to which it is tuned.*

I. INTRODUCTION

THE doublet antenna is known to have certain potential advantages in radio reception, especially in the reduction of noise from some causes. There are special problems in the design of doublet antenna systems to realize these advantages without exceeding practical limits on the size of the antenna and the nature of the associated apparatus. This paper deals with the problems of design and testing, and describes a complete system in which the desired advantages are realized.

In general, a doublet antenna system comprises four components which are all essential unless some of the functions are performed by virtue of inherent relations among the other parts. These components are (1) a balanced horizontal doublet, (2) an antenna-to-line switching filter for coupling the doublet to the transmission line, (3) a balanced transmission line for conducting the signal from the location of the doublet to the location of the receiver, and (4) a line-to-receiver shielding filter for coupling the line to the radio receiver. The method of de-

* Decimal classification: R320. Original manuscript received by the Institute, June 15, 1936. Presented before Rochester Fall Meeting of the I.R.E., November 20, 1935.

sign to be described minimizes the number of essential relations among the components and thereby secures the maximum simplification of the method of design.

The doublet antenna system must have signal-collecting ability at least comparable with that of a simple single wire antenna, and in addition must have advantages sufficient to justify its added complication. The advantages reside in the reduction of noise from certain near-by causes and in the predetermined directivity of the doublet as compared with the random directivity of ordinary single wire antennas. The noise problem and the related behavior of doublet antenna systems have been treated very well by J. G. Aceves.[1] The directivity depends mainly on the doublet, provided the other components are designed to have negligible effect on the signal-collecting properties of the system.

The reduction of noise depends primarily on the location of the antenna as far as possible from sources of local noise. This and the horizontal symmetry of the doublet minimize the pickup of local disturbances, which radiate mainly vertically polarized waves. The other components must then be designed not to couple the local noise to the receiver, either directly or by way of the antenna. Such local noise is generally picked up directly by the power circuit of the receiver, the lead-in wires, or the ground connection. The location of the receiver and lead-in wiring usually is not under control of the designer. The ground connection is preferably located near the antenna so as to minimize noise pickup, and it is utilized only at frequencies lower than the range of efficient operation of the antenna as a doublet.

The frequency range of operation is taken to be 0.54 to eighteen megacycles, the so-called "all-wave" range according to present standards. The antenna is operated exclusively as a doublet at six to eighteen megacycles, the frequency range of the most interesting weak signals and therefore the most severe noise problem. Mixed balanced and unbalanced operation is employed in the vicinity of five megacycles. At lower frequencies, it is operated as an unbalanced capacitive antenna.

The horizontal doublet[2] is most sensitive to horizontally polarized signals from the directions perpendicular to its length. Short-wave signals from distant points are generally elliptically polarized with horizontal components at least as great as the vertical components. The

[1] J. G. Aceves, "Problems of all-wave noise-reducing-antenna design," *Proc. Radio Club Amer.*, vol. 12, pp. 39–47; November, (1935).

[2] A horizontal loop antenna balanced to ground is the ideal expedient for nondirective reception of horizontally polarized waves. This was not employed because of the greater difficulty of installation in such form as to have sufficient signal-collecting ability.

directive curve in the horizontal plane has the simple form of a figure eight, since the length of the doublet to be described is less than the minimum wave length in the operating frequency range.

The unbalanced antenna is most sensitive to vertically polarized signals. It is nondirective in the horizontal plane, since the lead-in is connected at the center and the length is less than one half the minimum wave length in the frequency range of such operation. The lower frequency signals have greater vertically polarized components, so that unbalanced operation is in order.

The desirable properties of the antenna with respect to directivity are maintained by properly damping any unbalanced currents induced in the line. This damping may be secured by properly terminating the line to ground for unbalanced currents at either end, but preferably at the antenna end where the ground connection would be relatively free of noise pickup.

II. The Doublet Antenna

The antenna is designed with attention to its balanced operation as a doublet, since its unbalanced operation is much less critical as to dimensions. The design is based primarily on the measured impedance characteristics, relying on the published theoretical studies for the factors affecting the signal-collecting properties. The theoretical directive curves should be valid because the balanced operation is nearly independent of ground connections and associated apparatus.

The doublet should be installed with its length perpendicular to the direction of arrival of the signals of greatest interest. The directivity is an important advantage if so utilized. The least favorable reception in the direction of the length is limited to slight pickup of the vertically polarized component, depending on the proximity of the doublet to the ground. The sensitivity to the horizontally polarized components of low angle signals from the perpendicular directions is substantially proportional to the height above ground, as well as to the length of the doublet. The sensitivity to the vertically polarized components of low angle signals from the end directions is proportional to a sinusoidal function of the length and inversely proportional to the height. Therefore the probable directivity effective under ordinary conditions increases with the height and is not necessarily excessive in case the doublet is installed in an unfavorable direction at a moderate height.

Fig. 1 is an impedance curve whose general properties are to be discussed as the basis of designing the doublet for the present purpose. These curves were measured at the doublet terminals by means of

equipment designed especially for this purpose. The doublet is connected in parallel with the balanced tuned output circuit of a push-pull amplifier operating at the frequency of observation, and the amplifier is retuned to resonance by variation of capacitance. The change of capacitance and the reduction of output voltage are respectively used to evaluate the shunt susceptance and conductance components of the doublet admittance. Similar methods have been devised independently by other workers.[3]

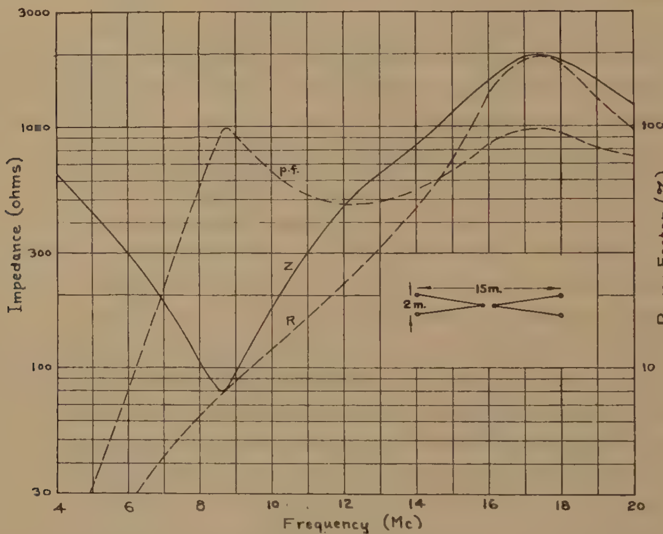


Fig. 1—Impedance of the fifteen-meter double V doublet.

The doublet must be coupled to the line by a fixed network which cannot be tuned to each operating frequency. Therefore any reactance of the doublet limits the signal current and the resulting efficiency of operation. The impedance ideally adapted for coupling with a transmission line is a uniform pure resistance. The departure from this ideal is indicated by the variation of impedance and power factor, shown in Fig. 1. The power factor curve is the best single indication of the degree of approximation to the ideal, because its shape determines approximatingly the shape of the impedance curve, whereas the converse is not the case at frequencies below the fundamental.

The so-called "double V" doublet with two wires on each side was chosen in preference to a simple single wire doublet of equal length on the basis of their respective impedance characteristics. The curves of

[3] E. Bruce, A. C. Beck, and L. R. Lowry, "Horizontal rhombic antennas," Proc. I.R.E., vol. 23, pp. 24–46; January, (1935).

Fig. 1 were observed on a fifteen-meter double V doublet of the shape and dimensions shown in the diagram. The double V doublet has the same minimum impedance (eighty ohms) as the simple doublet and only half the maximum impedance (2000 instead of 4000 ohms). The fundamental frequency of the double V doublet is about six per cent lower (8.6 instead of 9.2 megacycles) and the effective length is greater as affecting the radiation resistance and the power factor. The lowest operating frequency, as determined by the minimum permissible value of the power factor, is about fourteen per cent lower for the double V doublet. In other words, a simple doublet for the same frequency range would have to be longer by about sixteen per cent or 2.4 meters, and still would have twice the ratio of variation of impedance.

The advantages of the double V doublet over the simple doublet of the same length may be regarded as caused primarily by about thirty per cent reduction of the characteristic resistance and therefore of the mean impedance, and secondarily by the increase of effective length and therefore of the induced signal voltage.

There is a theorem relating to antennas, which is relevant to the problem of designing a circuit to couple an antenna with other apparatus. A nondissipative antenna operating at frequencies less than the fundamental is capable of delivering an amount of received signal power which is nearly independent of its dimensions. The power is limited by the induced signal voltage and the radiation resistance. The former varies in direct proportion to the effective height or length and the latter in proportion to the square of the same dimension. The square of the voltage divided by four times the resistance is therefore independent of the dimensions, and is equal to the maximum power which the antenna is theoretically capable of delivering to a load circuit. The delivery of the maximum power requires power factor correction by tuning the circuit to the signal, as well as impedance matching with the load circuit. The larger the antenna, the less critical is the tuning as to both reactance and resistance of the tuning elements.

At frequencies greater than the fundamental, a nondissipative antenna has relatively large values of the power factor caused by radiation resistance. The power factor correction is therefore less important, but the directivity is more critical. It is still desirable to minimize the reactance components of the antenna impedance to minimize the demand for power factor correction.

The connection in parallel of several equal wires not too close together, in place of a single wire, is a practical expedient for decreasing inductance and increasing capacitance, thereby reducing the reactance components of the antenna impedance. The average power factor is

increased and the impedance variation with frequency is reduced. The length of the several wires may be decreased with increasing number of wires if the effective length and the radiation resistance are to remain the same.

The doublet impedance characteristics make it impossible to secure the ideal impedance matching and power factor correction in coupling the doublet with a circuit of uniform resistance such as a transmission line properly terminated at the other end. The loss caused by the failure to realize these ideal conditions is defined as the "transition loss" in the art of transmission networks.[4]

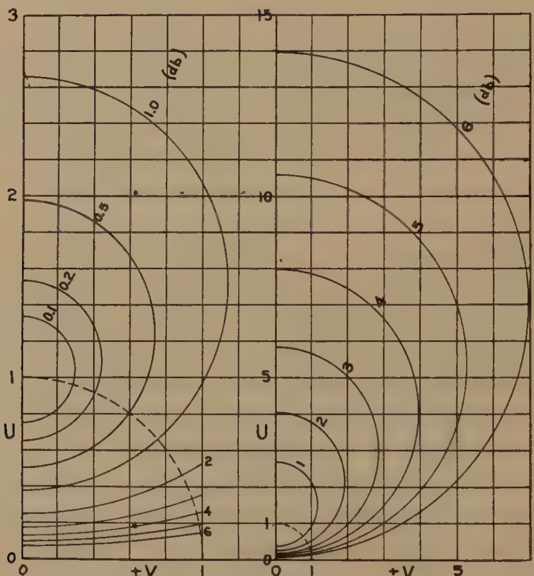


Fig. 2—Chart of transition loss (decibels) at the junction of impedance ($Z = R + jX$) and pure resistance (R_0) whose ratio (or its reciprocal) is represented by the co-ordinates ($U + jV$).

Fig. 2 is a chart which was prepared as an aid in evaluating the transition loss at the junction of two impedances, of which one is a pure resistance (R_0) and the other is, in general, a complex impedance ($Z = R + jX$). The ratio of these two impedances (or its reciprocal) is equal to $U + jV$, the co-ordinates of the chart. It is immaterial which impedance is in the numerator of the ratio. The circles are drawn with the loss as the variable parameter, so that the loss can be read directly for any impedance ratio within the range of the chart.[5]

[4] K. S. Johnson, "Transmission Circuits for Telephonic Communication," pp. 43–46, (1927); definition of transition loss.

[5] H. A. Wheeler, "Transition loss chart," *Electronics*, vol. 9, p. 27; January, (1936).

Fig. 3 illustrates the use of the transition loss as a measure of the mismatching of the doublet impedance with the resistance of a line. The dotted curve is based on the doublet impedance curve of Fig. 1 and a line of 500 ohms resistance. The solid curve is based on the same line but the doublet is loaded with parallel inductance which tunes the antenna to six megacycles, and thereby causes the transition loss to be less between five and nine megacycles and not much greater at higher frequencies. This is indicative of the benefit obtainable by careful design of the circuit coupling the doublet with the line. Above 5.5 megacycles, the loaded doublet is subject to less than three decibels loss relative to the ideal condition of tuning the antenna at each operating frequency and optimum coupling to the line with nondissipative circuits.

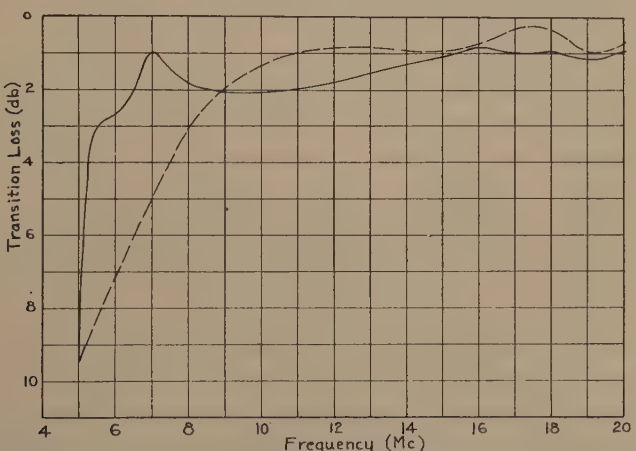


Fig. 3—Transition loss at the junction of the fifteen-meter double V doublet and a 500-ohm line (dotted curve), and at the same junction loaded with parallel inductance (solid curve).

It is apparent that the marginal advantage of further improvement of the antenna is not sufficient to justify much greater length or weight. The theoretical advantage of the double V doublet over a simple doublet of the same length varies from about two decibels at frequencies below the fundamental to about one decibel at the fundamental and higher frequencies. The practical advantage is greater because fewer circuit elements are required in loading the fifteen-meter double V doublet for the required frequency range.

III. The Antenna Filter

The antenna-to-line filter serves the purpose of coupling the antenna to the line most effectively for balanced-doublet operation in the higher band of five to eighteen megacycles and unbalanced operation in the lower band of 0.54 to five megacycles. Therefore it is a

switching filter including impedance matching transformers. It is especially important that unbalanced antenna currents at frequencies in the high band should not be coupled to the line, because one of the main purposes of the system is to minimize the disturbing effects of such noise currents.

Fig. 4(a) shows the general arrangement of the antenna system, with the circuit details of the antenna filter. The upper and lower dotted rectangles include respectively the high band and low band transformers and associated elements. The transmission line is coupled

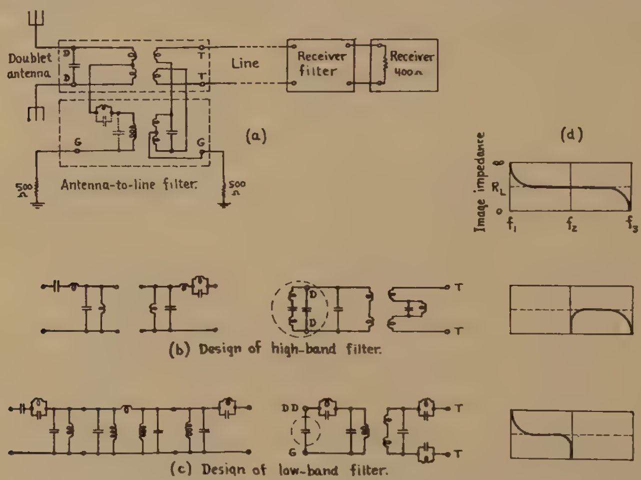


Fig. 4—The doublet antenna system and the design of the antenna-to-line filter.

with the receiver by the receiver filter. The receiver and the receiver filter are designed to terminate the line by impedance substantially equal to its characteristic resistance at the frequency to which the receiver is tuned. Therefore the line can be regarded as a uniform resistance load on the antenna filter. Also the performance of the system therefore depends on the length of line, only to the extent of the line attentuation.

The ground connection of the antenna circuit is likely to be so long as to resonate within the operating frequency range and therefore to have abnormally high impedance at certain frequencies. It is desirable to have this impedance as low as possible. Its maximum values may be reduced at the expense of increasing the minimum values, by adding damping resistance of about 500 ohms at the ground end of the ground connection. The impedance to ground is then nearly uniform at about 500 ohms.

A separate similar ground connection may be connected to a center tap at the antenna end of the line, in order to terminate the line to ground properly for unbalanced currents. This makes the line aperiodic with respect to unbalanced signal pickup, while the balanced pickup is minimized by the close spacing of the line wires. In this manner, the effect of the line on the signal-collecting properties of the system is minimized.

The separation of the two ground connections is desirable to minimize the coupling of unbalanced currents from the line to the antenna circuit. If only one connection is to be used, it should be that of the antenna circuit, and the antenna end of the line is then left ungrounded.

If a quiet ground connection is not available in the vicinity of the antenna, the primary and secondary ground terminals *G*-*G* may be connected together and not to ground. The line then serves as ground connection or counterpoise, depending on whether there is a ground connection to a center tap on the primary side of the receiver filter. This expedient is likely to decrease the amount of noise reduction very much for operation in the low band, and slightly for operation in the high band.

The design of the high band filter is indicated in Fig. 4(b). It is based on the two band-pass half sections shown on the left.[6] The first half section is of the "constant-k" type. The second is of a derived type whose series arm includes parallel elements resonant at a frequency slightly below the band. These half sections are reduced to an equivalent circuit including a transformer with balanced primary and secondary circuits. The input terminals of the first half section are connected together for the transformation. The four elements in the dotted circle have an impedance curve similar to that of the doublet, so they are to be replaced by the doublet. The parallel elements in the secondary circuit are to be replaced by the low band filter.

The design of the low band filter is indicated in Fig. 4(c). It is based on the two half sections and one whole section of band-pass filter shown on the left. The half sections are of derived types whose series arms include parallel elements resonant at frequencies slightly above the band. The whole section is of a simple type adapted for transformer substitution. These are reduced to an equivalent circuit including a transformer with unbalanced primary and balanced secondary circuits. The input terminals of the first half section are connected together for the transformation. The condenser in the dotted circle is to be replaced by the doublet connected as an unbalanced capacitive

[6] T. E. Shea, "Transmission Networks and Wave Filters," pp. 315–318, (1929).

antenna. The parallel elements on either side of the secondary circuit are to be replaced by the high band filter.

The connection together of the input terminals in the derivation of the high band and low band filters is preferable to the addition of terminating resistance. Such resistance would yield some advantage in causing the filter to present more nearly uniform impedance at the line terminals. This advantage is more than counteracted by the dissipation caused by such resistance, and the resulting attenuation of the signal. Wherever the input terminals are to be joined, it is advisable

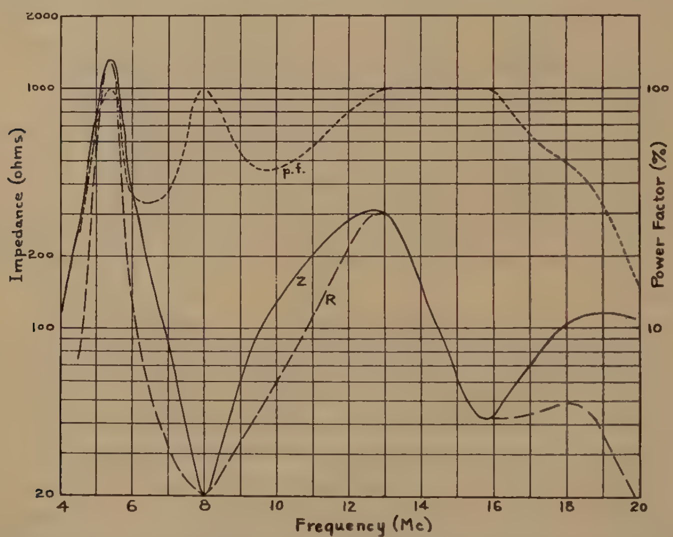


Fig. 5—Impedance presented to the transmission line by the antenna filter connected with the antenna.

to choose a filter circuit such that its image impedance across these terminals is substantially zero at the cutoff frequencies. This rule is followed in Figs. 4(b) and 4(c). The short-circuit termination then matches the image impedance at the cutoff frequencies, whose identity is thereby preserved.

Fig. 4(d) shows the image impedance relations on the secondary side of the antenna filter. The reactance components are approximately cancelled in both bands, so only the resistance components are shown. The resultant image impedance is nearly uniform over most of the operating frequency range, even in the vicinity of the boundary between the low and high bands. This result is secured by the careful choice of the types of secondary half sections and the values of their parameters. The relation between the low band and high band filters in practice is not

as critical as the diagrams would seem to indicate, because of the dissipation and the small number of sections in each filter.

The number of elements in the high band filter is the minimum consistent with the insertion of a transformer and the inclusion of the doublet as part of the filter. The number of elements required in the low band filter is greater in order to secure sufficient attenuation in the high band and to cover the low band without requiring too close to unity coupling in the transformer. The trap in the low band primary circuit is necessary to secure adequate attentuation of unbalanced currents at six megacycles, near the boundary between the bands.

Fig. 5 shows the high band impedance curve which the antenna filter presents to the line, with antenna and ground connected for normal operation. The geometric mean impedance is approximately equal to 115 ohms, the characteristic resistance of the line for which this filter was designed.

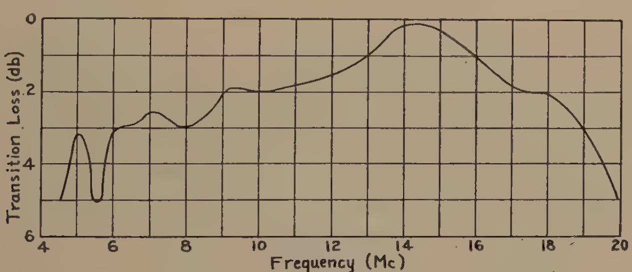


Fig. 6—Transition loss at the junction of the antenna filter and the line.

Fig. 6 shows the transition loss at the junction of the antenna filter and the line. It is less than two decibels between nine and eighteen megacycles, where greatest sensitivity is needed, and less than three decibels between six and nine magacycles.

The transition loss from doublet to filter is slightly greater by an amount depending on the low band filter and on the dissipation of the high band filter. To this must be added the filter attenuation to evaluate the transition loss from antenna to line.

IV. The Transmission Line

The transmission line should comprise a pair of conductors balanced to ground and located fairly close together. Transposed or parallel spaced conductors cause negligible attenuation but offer some difficulty of installation. Twisted or untwisted pairs of insulated conductors close together are easy to install but cause substantial attenuation at the higher frequencies, especially when wet. The system being de-

scribed was designed for a 115-ohm line of the latter type. Such a pair should have high grade rubber insulation for operation at six to eighteen megacycles. Such lines of moderate weight, in the dry condition, have minimum attenuation per hundred feet of about one decibel at six megacycles and two decibels at eighteen megacycles. Most samples tested dry have greater attenuation, and much greater when tested wet. The pair is usually covered with a lacquered braided sheath to protect the rubber. This sheath adds little to the attenuation when dry, but when wet it adds about three decibels per hundred feet. Also it delays the drying after exposure to moisture. It is desirable, if possible, to develop a rubber-covered pair which will stand exposure without any braided covering.

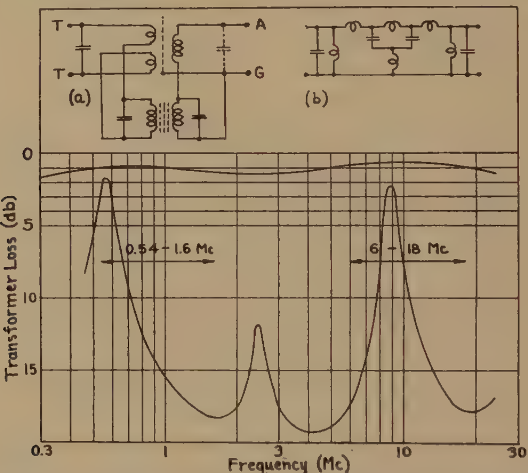


Fig. 7—The line-to-receiver filter (a), a network having the same filter properties (b), and the observed attenuation in the receiver filter with normal termination (upper curve) or abnormal termination (lower curve).

V. The Receiver Filter

The line-to-receiver filter serves to couple the balanced line to the unbalanced input circuit of the radio receiver, without coupling thereto any unbalanced currents in the line. Therefore this filter must pass the entire frequency range and must include a transformer with adequate shielding. Incidentally, the total capacitance between primary and secondary circuits should be minimized because it couples unbalanced noise currents from receiver to line to antenna.

Fig. 7(a) shows the cuircuit of the receiver filter. Separate high band and low band transformers are employed because it is impractical to secure sufficiently close to unity coupling in a single radio-

frequency transformer to operate over such a wide range of frequency. The high band transformer has between its coils a capacitive shield comprising a sleeve of copper foil, insulated at the lap joint to prevent inductive shielding. The low band transformer is made of two concentric multilayer coils arranged so that one of the extreme layers of the secondary serves as a capacitive shield from the primary. An open core of iron dust is used to permit more space between the latter coils while retaining the required coefficient of coupling.

This double transformer filter was designed in accordance with formulas derived by an application of the equivalent lattice method of filter synthesis.[7] It is found theoretically to have the properties of a continuous band filter if the circuit elements have the proper relations. The high band filter alone passes approximately the upper third of the frequency range, the low band filter the lower third, and both together the middle third. The double transformer filter has less attenuation than a filter passing the same band by means of a single transformer of very close coupling.

Fig. 7(b) shows for comparison a ladder-type filter whose performance characteristics are identical with those of Fig. 7(a). The four elements in the middle of the ladder comprise an "all-pass" section which has little effect on the attentuation. Therefore the attenuation in the double transformer filter differs very little from that in a single transformer filter, although the latter requires much greater coefficient of coupling.

The receiver filter was computed and designed to pass the band of 0.54 to twenty-five megacycles, the upper cutoff frequency being sufficiently high to assure minimum attentuation at six to eighteen megacycles. The upper curve of Fig. 7 shows the loss in this filter relative to an ideal transformer operating between the same terminal resistances of 115 and 400 ohms, respectively.[4] The loss does not exceed 1.5 decibels over a frequency range much greater than the operating range of 0.54 to eighteen megacycles. The lower curve shows the corresponding loss with much higher values of terminal resistance, in order to identify the peaks of the curve at the critical frequencies of the design.

VI. The Over-All Performance of the System

Measurement of the loss of the entire system is desirable in order to check the performance of all components operating together. Such a test requires a dummy antenna in which a signal can be introduced in

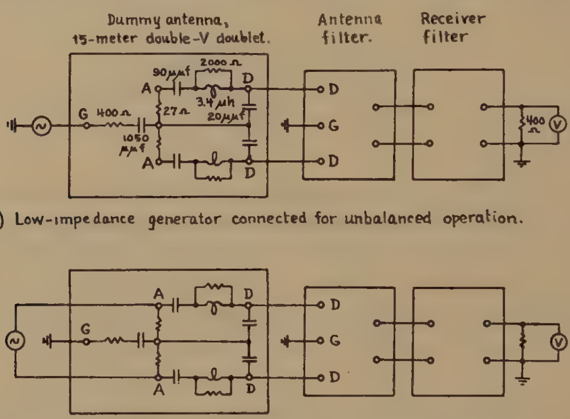
[7] H. W. Bode, "A general theory of wave filters," *Jour. Math. and Phys.*, vol. 13, pp. 275–362; November, (1934); summary, *Bell Sys. Tech. Jour.*, vol. 14, pp. 211–214; April, (1935).

[4] K. S. Johnson, *loc. cit.;* definition of transformer loss.

such a manner as to simulate either balanced or unbalanced operation. Also the dummy antenna must present to the antenna filter both balanced and unbalanced impedance approximating that of the antenna over the entire operating range of frequency.

Fig. 8 shows the complete arrangement for making over-all tests. The dummy antenna is designed to meet all the requirements. The other components of the antenna system are connected normally. The output voltage is measured across a dummy load of 400 ohms resistance to simulate the receiver.

The balanced elements of the dummy antenna simulate the factors which are essential in determining the doublet impedance. The con-



(a) Low-impedance generator connected for unbalanced operation.

(b) High-impedance generator connected for balanced operation.

Fig. 8—Special dummy antenna connected for measurement of over-all loss in the antenna system.

densers across the terminals *D-D* simulate the capacitance near the terminals at the center of the doublet, in which there is induced negligible signal voltage. The condensers next to the terminals *A-A* simulate the main part of the doublet capacitance, in which is induced the signal voltage. All the elements are proportioned so that the impedance curve observed between the terminals *D-D* very closely approximates that of Fig. 1 in all respects.

The unbalanced impedance of the dummy antenna is that between the ground terminal *G* and the two terminals *D-D* tied together. The condenser in the ground lead of the dummy antenna has such a value that the unbalanced capacitance of the dummy antenna at frequencies much less than the fundamental is equal to the capacitance between the doublet wires connected together and the ground lead connected to ground. The resistor in the ground lead of the dummy antenna simu-

lates the damping resistance at the base of the ground lead, but has a lesser value because it carries all the current whereas the damping resistance carries somewhat less than the maximum current in the ground lead.

Fig. 8(a) shows the connection of the signal generator for unbalanced operation, in series with the unbalanced impedance of the dummy antenna. Fig. 8(b) shows the connection for balanced operation, in which the signal generator is required to introduce a balanced voltage in the dummy antenna. A signal generator of high impedance is connected to the balanced input terminals *A-A* across the center-

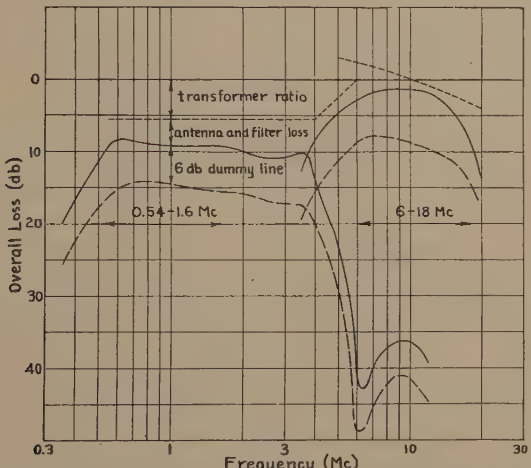


Fig. 9—Over-all loss in the system, for unbalanced operation of antenna at the lower frequencies and balanced operation at the higher frequencies.

tapped resistance. The signal input voltage is taken as the voltage across *A-A* with the arms *A-D* both on open circuit. This method simulates the normal balanced operation of the doublet, in that the signal voltage appears in series with those elements representing the main part of the doublet, and not in series with the capacitance near the center of the doublet.

Fig. 9 shows the over-all loss of voltage from dummy antenna to dummy load under various conditions of operation. The low band curves are for unbalanced operation and the high band curves are for balanced operation. The solid upper curves show the minimum loss for zero length of line. The dotted lower curves show the corresponding loss for a dummy line of six decibels attenuation. The dummy line comprises merely a balanced symmetrical attenuator section of image impedance equal to the characteristic resistance of the line for which the

circuits are designed. The upper and lower curves should have uniform vertical separation equal to the attenuation of the dummy line. The approximation to this relation indicates the closeness of matching the antenna and receiver filters with the line.

The components of the over-all loss are indicated for reference. The "transformer ratio" is the product of the ratios involved in the design of antenna and receiver filters, and represents only loss of voltage as distinguished from loss of power. The transformer ratio has a transition in the region of five megacycles, the nominal boundary between the low and high bands. The "antenna and filter loss" is caused by dissipation and reactance of the antenna and both filters. About half of the latter is caused by the effective resistance of the ground lead.

The over-all loss is shown plotted in terms of a voltage ratio expressed in decibels. This loss should be compared with the minimum loss theoretically possible if the antenna were tuned to each signal and ideally coupled to the 400-ohm load simulating the receiver. The resistance component of the dummy antenna unbalanced impedance is 400 ohms in series with the signal generator, so that the minimum possible loss of voltage is six decibels for unbalanced operation at the lower frequencies. The resistance effectively in series with the signal generator for balanced operation is variable with frequency. The corresponding theoretical curve of minimum loss is the dotted curve above the high band curves of Fig. 9. This curve corresponds to zero transition loss from doublet to receiver, and therefore represents the best performance theoretically obtainable with this antenna operated as a doublet. It is noted that the observed loss over the entire frequency range is only between one and five decibels greater than the theoretical minimum.

The loss added by the transmission line is the attentuation of only the necessary length of line, because the length is not critical when the line is properly loaded, as in this design.

An important characteristic of the system is the great attenuation of unbalanced signals at frequencies in the high band. The relative attenuation is more than thirty-two decibels in the range of six to eighteen megacycles where refinement of balanced operation is essential to maximum reduction of noise. There is a problem to secure uniformly good performance in the vicinity of the boundary between low and high bands, together with sufficient attenuation of unbalanced signals in the high band. This result was secured with the aid of the six-megacycle trap in series with the primary circuit of the antenna filter, shown in Fig. 4.

VII. The Receiver Input Circuit

It is desirable to design the input circuit of the receiver in such a manner that it is adapted for connection with either a simple antenna

or a doublet antenna system. The former is ordinarily assumed to have impedance approximating 200 micromicrofarads capacitance at frequencies less than 1.6 megacycles and 400 ohms resistance at greater frequencies. The doublet antenna system is designed to present at the output terminals of the receiver filter, impedance having a mean value of 400 ohms resistance at frequencies over the operating range of 0.54 to eighteen megacycles.

Fig. 10 shows an example of an input circuit adapted for connection alternatively with either a capacitive simple antenna or a resistive antenna system. The main requirements are effective coupling between either antenna and the tuned circuit, and similar effects of either antenna on the resonant frequency of the tuned circuit. There is a real

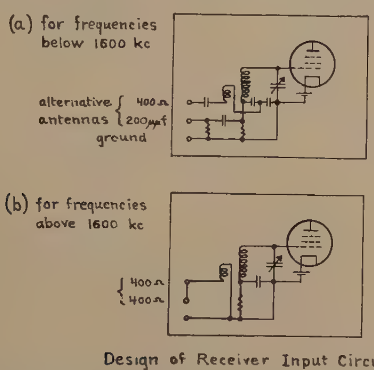


Fig. 10—Receiver input circuit adapted for connection alternatively with ordinary antenna (200 micromicrofarads or 400 ohms) or doublet antenna system (400 ohms).

problem only at lower frequencies where there is most difference of the properties of the two types of antennas.

The circuit of Fig. 10(a) for operation at 0.54 to 1.6 megacycles, has two antenna terminals connected with different primary circuits coupled differently to the same tuned secondary circuit. Many alternative forms of coupling are available for this purpose.[8] The design shown is based on simple coupling for the capacitive antenna, furnished by a relatively large condenser common to the primary and secondary circuits. (The added filter comprising two resistors and a condenser is for direct-current and low-frequency isolation and does not appreciably affect the radio-frequency operation.) Such coupling gives substantially uniform voltage ratio from the antenna to the secondary circuit, as well as very little effect of the antenna on the resonant frequency.

[8] H. A. Wheeler and W. A. MacDonald, "Theory and operation of tuned radio-frequency coupling systems," Proc. I.R.E., vol. 19, pp. 738–803; May, (1931).

The coupling circuit for the resistive antenna system includes both capacitive and inductive self-reactance in the primary circuit, as well as both kinds of mutual reactance between primary and secondary circuits. Therefore there are four degrees of freedom in the design of the coupling circuit, which can be utilized to satisfy four conditions. Two conditions are that the effect on the resonant frequency should be the same as that of the capacitive antenna circuit, at frequencies near both limits of the tuning range. The other two conditions are that the coupling should be optimum between the tuned circuit and a 400-ohm resistive dummy antenna, at frequencies near both limits of the tuning range. The latter condition makes the input impedance of the receiver at resonance substantially equal to 400 ohms resistance. As a result, either of the two input circuits may be connected with its respective antenna, with assurance of normal operation over the tuning range.

Fig. 10(b) shows the simple input circuit adapted for either type of antenna at frequencies above 1.6 megacycles, where either type presents a mean value of impedance approximating 400 ohms resistance. This coupling circuit has two degrees of freedom, the primary self-inductance and the mutual inductance. These are utilized to secure optimum coupling between the tuned secondary circuit and the 400-ohm resistive dummy antenna, at frequencies near both limits of the tuning range. The input impedance of the receiver at resonance is thereby made equal to 400 ohms resistance, as desired for terminating either the simple antenna or the receiver filter of the doublet antenna system.

VIII. Conclusion

By following the principles outlined, a doublet antenna system is secured which is dependable in operation and has the maximum of simplicity and of freedom from incidental effects of the length of the transmission line. Balanced operation of the doublet antenna at the higher frequencies and of the line at all frequencies secures the maximum reduction of noise which is consistent with maintaining the maximum signal-collecting ability. The latter requires unbalanced operation of the antenna at the lower frequencies. The balanced operation of the horizontal doublet is unavoidably directive, but the directivity is substantially independent of frequency and is predetermined so that it can be used to advantage.

Acknowledgment

The authors desire to acknowledge the helpful co-operation of their colleagues, especially the valued assistance of Mr. R. E. Sturm, whose ingenuity and perseverance in performing the great amount of experimental work contributed very much to the value of the results.

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(9) W. W. MacAlpine, U. S. Patent No. 2,021,734, November 19, (1935); antenna-to-line and line-to-receiver double transformers.

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ELECTRON BEAMS AND THEIR APPLICATIONS IN LOW VOLTAGE DEVICES*

By

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Summary *—A study has been made of the segregation into beams of the space current in devices of the over-all size of commercial receiver tubes and at potentials less than 300 volts. Electrode coatings of luminescent material were used to make the beam traces apparent on any or all electrodes. Qualitative relationships between beam formations and relative electrode potentials are stated. Space currents of a few milliamperes have been concentrated into beams less than 0.010 inch wide in simple structures. Special control grids associated with conventional cathodes have been found to combine good space current control with effective beam formation. These effects together with control of beam width and direction can be obtained in a single device.*

Simple structures are used to segregate the electron discharge from a single cathode into a few or many similar electron beams. Between such structures and the outermost electrodes are placed perforated positive electrodes with their openings disposed to receive and pass the electron beams. Such electrodes are found to receive as little as two per cent of the current computed from their projected area. Such perforated electrodes include conventional type grids.

The relationships between beam widths and electrode potentials have been found to be such that mutual volt-ampere relations can be radically different from those hitherto utilized. Linear, saturation type, or special volt-ampere relations can be obtained. Individual electrode volt-ampere relations can be altered by utilizing the action of the field of such an electrode to change the width and direction of the beam impinging upon that electrode independently of interelectrode coupling. A variety of negative conductance devices have been made on this principle.

I. INTRODUCTION

THE high vacuum devices in which electron beams have been used previously, such as the X-ray tube and the cathode-ray oscillograph, have employed, for the most part, small space currents, single beams of circular cross section produced by essentially spherical electrostatic or magnetic lens systems, and high electrode potentials. Attempts to take advantage of the properties of beams for other purposes have usually resulted in structures more or less like those employed in the above devices and evidently derived therefrom. Such structures, as compared with receiving tubes and small power tubes, tend to be expensive to make, large in size, inefficient in their use of cathode power, and inadequate in their ability to carry current.

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The present investigation represents a different approach to the subject, and was begun as the result of an observation made more than ten years ago on a commercial form of triode whose anode had been coated with luminescent material. With a certain ratio of potentials on grid and anode there appeared on the latter narrow, sharp-edged luminous lines opposite the openings between grid wires, indicating a nearly complete segregation of space current into beams. The space current so segregated was many milliamperes at anode voltages of two or three hundred and the widths of the luminous bands could be controlled by the electrode potentials.

These observations suggested that electrodes of various kinds, located in the noncurrent carrying spaces between beams and used for control purposes, might have a high input impedance even though operated at positive potentials, and that screening electrodes similarly placed might receive and waste very little space current. It was also thought that the variation of beam width might make possible, by redistribution of space current between various positive electrodes suitably placed, transconductance and conductance characteristics of kinds unobtainable in devices utilizing the space-charge phenomenon alone.

Such expectations have been fulfilled in a variety of devices which are characterized by compactness, relative simplicity, high current carrying ability at low potentials, the use of rectangular section beams produced by cylindrical lens systems, the use of several beams from a common cathode, and structural forms adapted to assembly methods more nearly like those for amplifier and small power tubes than those for devices of the cathode-ray oscillograph type.

The devices here presented belong, with one exception, to a class of devices in which the advantages of segregation of the discharge into beams and of variation of beam width and focus are utilized. Two other classes of devices dependent upon the deflection of the median planes of rectangular section beams and upon the use of magnetic fields are excluded from this discussion.

The work along these lines was begun a number of years ago, and the elementary principles and potentialities of it were established. Later it was learned that work of a somewhat similar kind was being done in Germany by Knoll and Schloemilch.[1,2]

[1] M. Knoll and J. Schloemilch, "Electron optical current distribution in electron tubes with control electrodes," *Arch. für Elektrotech.*, vol. 28, pp. 507–516; August 18, (1934).

[2] M. Knoll, "Amplifying and transmitting valves considered as a problem in electron optics," *Zeit. für. Tech. Phys.*, vol. 51, no. 12, pp. 584–591, (1934).

II. Methods of Observation

In structures for the study of beam phenomena appropriate electrodes were coated with a thin and uniform layer of fine particles of a luminescent material such as willemite. Such a layer becomes luminous where and only where electrons of sufficient energy arrive at a sufficient rate. If the space current is divided into beams, there appears on a coated and bombarded electrode a luminous pattern corresponding to the cross section of the beams at the surface of the electrode. Observations of that pattern can often be made with the potential of the bombarded electrode as low as twenty volts. In cases where it was not clear from the luminous pattern of beam traces just what had happened to the cross section of the beam in transit to the target, a low pressure of gas and the consequent glow made it possible to observe the beam throughout most of its path. The pressure of the gas introduced for this purpose was always insufficient to cause observable change in the trace patterns and hence the observed beam paths were only slightly different from those in high vacuum. It will be seen from the data below that this method of study is of practical utility in the design of beam devices.

The first ten figures illustrate the beam-forming properties of simple structures in association with conventional forms and sizes of cathodes and certain elementary properties of the beams thus formed. The remaining figures show some typical devices for the utilization of such beam properties.

III. Observations

Fig. 1 shows scale sections of a triode with parallel plane electrodes and indicates the observed beam traces on the anode or No. 2 electrode at 250 volts for four different grid or No. 1 electrode potentials. The shapes of the equipotential surfaces in the grid openings are roughly indicated. With the grid at seventy-six volts positive, approximately geometrical shadows of the grid wires appear as indicated on the anode with temperature-limited space current. The grid is here approximately at its space potential, the calculated value of which is seventy-eight volts. A similar trace pattern was observed with space-charge-limited current with the grid at its correspondingly lower space potential. In both cases the equipotential surfaces are parallel planes except in regions very close to individual grid wires. In the same figure, the traces for twenty-two and five-tenths volts positive and for zero volts on the grid correspond, respectively, to focus of the electron beam on the anode and between the anode and grid. These two cases are for space-charge limitation of the current. The fourth section of the figure shows

the result of placing the grid above its space potential at 172 volts whereby the grid opening becomes a diverging lens and the traces of adjacent beams overlap, as indicated, on the anode.

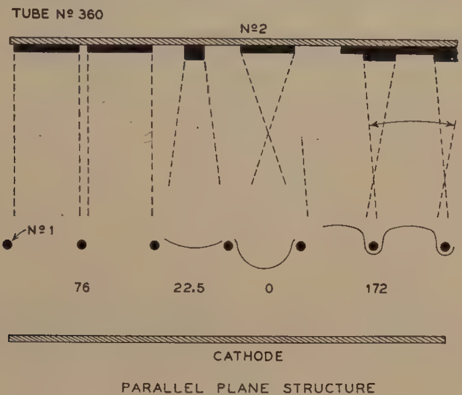

Fig. 1—Beam formation by grid openings.

No. 2 volts = 250

No. 1 volts	*Beam Focus*
0	Before No. 2
22.5	On No. 2
22.5–76	Beyond No. 2
76	At infinity
172	Virtual

The actual appearance of the beam traces for these four cases is shown in Fig. 2, which is a group of four photographs of the anode of the triode of Fig. 1 for the above four conditions. The preceding two

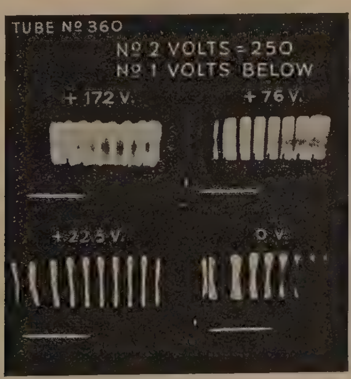

Fig. 2—Beam traces on an anode.

figures are representative of the manner of beam formation not only for the plane grid structure, but for a helical grid in a cylindrical structure at different but corresponding potentials. In the cylindrical case,

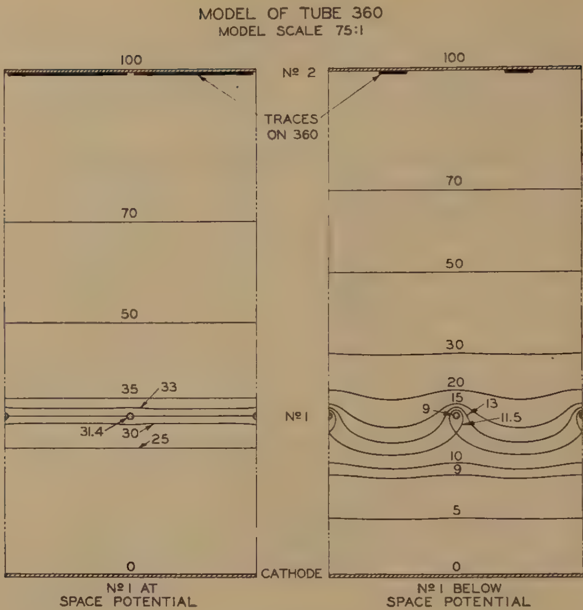

Fig. 3—Equipotential maps on scale model.

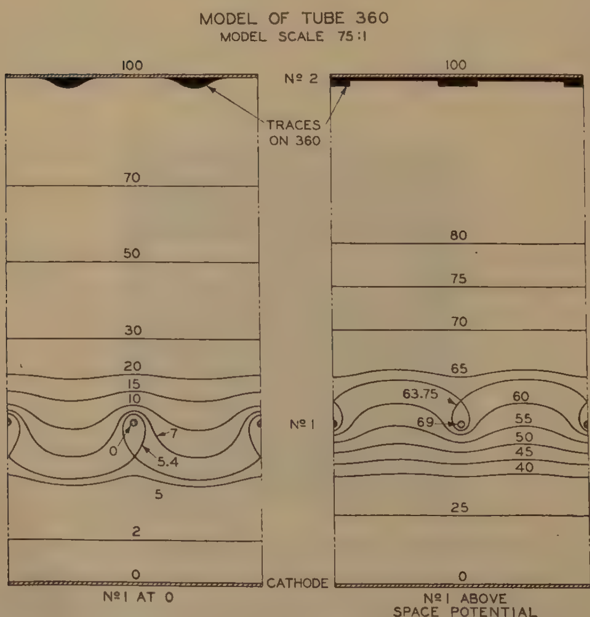

Fig. 4—Equipotential maps on scale model.

the traces for focus often have a width one tenth or less of the distance between traces and are very sharp edged.

Figs. 3 and 4 are maps of the equipotential surfaces obtained by means of a field-mapping apparatus on a 75-to-1 scale model of the triode of Figs. 1 and 2. The potentials are there shown as percentages of the anode potential; the grid potentials correspond in percentage to the actual potentials of Figs. 1 and 2. The field maps for the grid potentials of nine and zero per cent cannot, of course, correspond accurately to those existing in the observations of beam traces above, be-

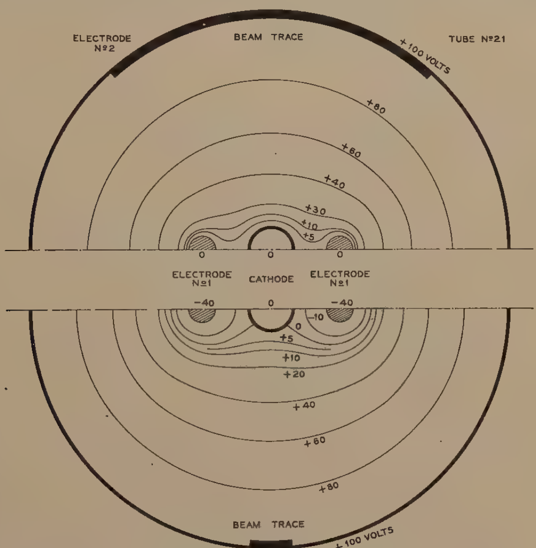

Fig. 5—Beams by a longitudinal element grid.

cause, as stated, those two particular observations were made for the space-charge-limited condition which is not simulated by the field-mapping apparatus. The two maps indicate the kind of field in the grid opening which constitutes a converging lens. The above figures and data illustrate the fact that the space current can be segregated into beams without the cutting off of emission from any part of the cathode.

Fig. 5 shows a cylindrical anode, or No. 2 electrode, a coaxial cathode, and two longitudinally placed wires constituting the beam-forming or No. 1 electrode. With the latter at forty volts negative bias, two narrow beams are formed. The anode trace of one of them is indicated. The location of the intersections of the zero equipotential surface with the cathode surface for that case indicates that a considerable part of the cathode is cut off as regards emission, unlike the case of the first-

mentioned device for similarly narrow beams. The map for zero bias of No. 1 electrode shown in the upper half of the figure indicates no cutoff of emission at the cathode.

In Fig. 6, a typical instance of the manner in which the width of the beam trace varies with anode potential is shown. The unlabeled electrode is the cathode in this and all following figures. For a positive

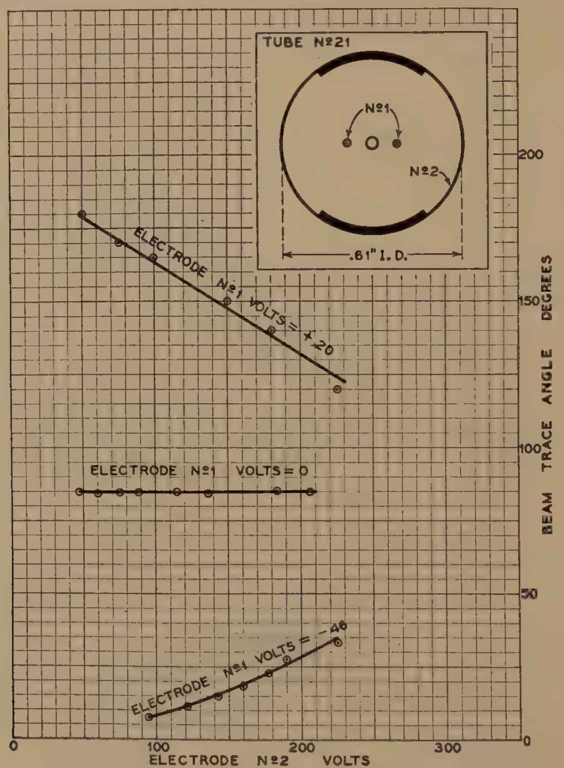

Fig. 6—Beam width and anode potential.

bias on the beam-forming electrode, an increase of anode potential causes a decrease of beam width, while for a negative bias the reverse is true. This illustrates a behavior which proves to be general except in such structures and at such potentials that a crossover or focus of the beam occurs before it reaches the anode. The constancy there shown of the beam width for zero bias of No. 1 electrode is typical of closely spaced structures for anode potentials above a certain value for each particular structure. At anode potentials lower than that value the traces are observed to increase in width in some cases, before they become invisible. There are several causes tending to spread the beam at

such low potentials, and it cannot be said at once which of these predominates.

Corresponding beam formations by longitudinal wires have been observed for numbers of wires from one to eight, and for a wide range of wire diameters and spacings from the cathode.

Fig. 7 shows a case in which the two wires are much larger than the cathode. It also shows the constancy of beam width when the ratio of the potential on No. 2 to that on No. 1 is kept constant. This illustrates another general behavior subject to qualification for low potentials as above.

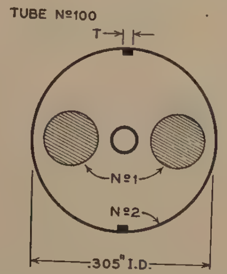

Fig. 7—Constant ratio of electrode potentials.

$$\frac{\text{No. 2 volts}}{\text{No. 1 volts}} = 3$$

No. 1 volts	*No. 2 volts*	*No. 1 ma*	*T (inches)*
−20	60	0.8	0.02
−30	90	2.0	0.02
−40	120	3.3	0.02
−60	180	7.2	0.02
−80	240	12.1	0.02
−100	300	15.5	0.02

Fig. 8 shows an example of the way in which beam width and current vary simultaneously with negative bias on a two-wire beam-forming electrode. Fig. 9, derived from Fig. 8, shows the constancy of the current-to-width ratio over a considerable range. It should not be concluded from this that the current per unit width is constant throughout the beam. Actually a "piling-up" of current at the sharp edges of a wide beam is often observed. No generalization of current-to-width ratio or of current distribution in beams can be made at present.

Such two-wire structures, while useful for beam formation alone, have a very poor control over the space current. In Fig. 10, however, a beam-forming structure No. 1 is shown which gives a transconductance more nearly of the conventional order and also a segregation into a rather narrow beam. The coated area of the cathode is about seventy-five per cent of that of an RCA-57 cathode of the same size. The struc-

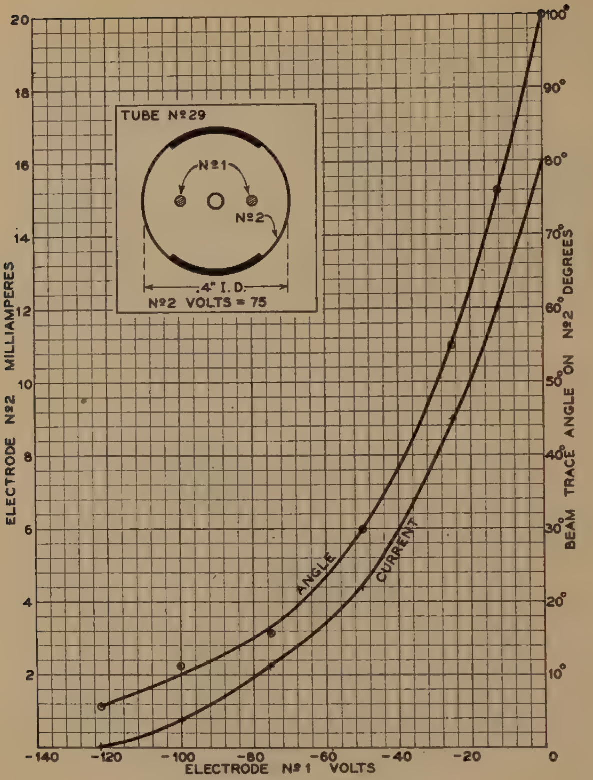

Fig. 8—Beam width and current.

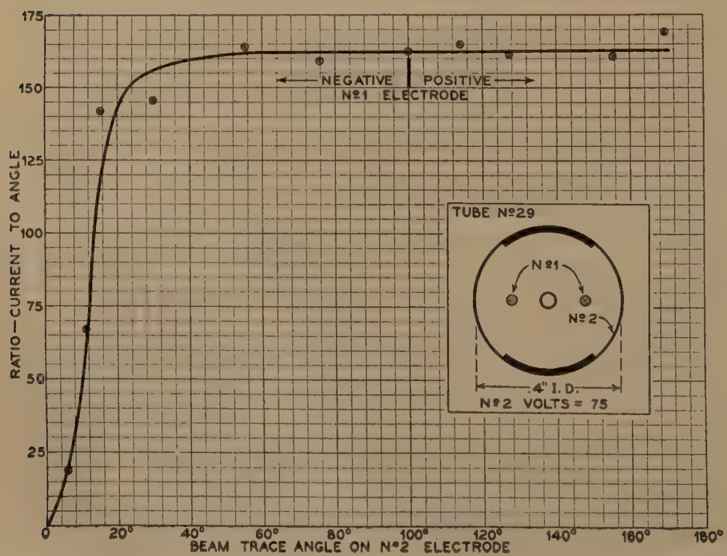

Fig. 9—Current-to-width ratio of a beam.

ture consists of a helical wire grid enclosed in a sheet-metal cylinder with a slit as indicated. The wires of the grid are thus transverse with respect to the longer dimension of the rectangular section beam formed by the slit and, while they cause the beam trace to be divided up into sections along that dimension, they do not seriously distort it in its shorter dimension.

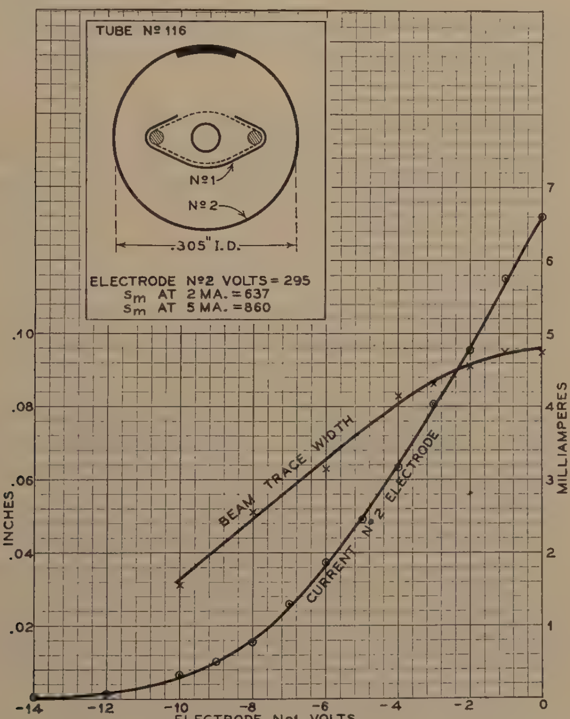

Fig. 10—Beam formation and normal transconductance.

IV. Discussion of Observations

From the above and many other observations, certain simple principles can be formulated which are sufficiently general in their applicability to constitute a valuable basis for the design of beam devices of the kind presented here. With reference to small triode structures operated space-charge limited and to anode potentials above a value which varies with the structure but which is usually of the order of twenty volts, the observations support the following statements:

(1) A nonemitting electrode operating at cathode potential and constituting a part of the cathode or intervening between it and the

anode determines, on the anode, trace patterns which are constant with varying anode potential.

(2) If the ratio of the potential of the intervening electrode to that of the anode is kept constant, the beam traces are also constant. This generalization, of which the preceding is a special case, is consistent with a corresponding mathematical generalization on electron paths where initial electron velocities are zero, as expressed by Langmuir and Compton.[3] The error caused by initial velocities has been too small to be observed in the small structures used except, as mentioned above, at low electrode potentials.

(3) If a thin electrode containing an opening is introduced between cathode and anode in such a way as to conform to a previously existing equipotential surface, the opening acts upon the electron stream through it as a converging or a diverging lens, according to whether the electrode is below or above its space potential, respectively. The opening has approximately no deviating effect upon the stream when the electrode is at its space potential. In the latter case a grid is observed to form on the anode a shadow pattern which is a geometrical projection of itself.

In many structures, the converging effect of an opening may be such, at suitable potentials, as to give a focus or crossover of the electron stream either at or before the anode as in Fig. 1. In other structures such as that of Fig. 5, the divergence of the beam entering the opening may be so great as to prevent the formation of such a focus at potentials less negative than those at which there is complete cutoff of the space current.

(4) In structures according to (3) and at such potentials that no focus occurs, it can be said that an increase of anode potential decreases the dimensions of the beam trace when the apertured electrode is positive and increases them when that electrode is negative. For the range of potentials in which one crossover of the beam occurs the reverse statement is true.

(5) For low positive, zero, or negative biases of the beam-forming electrode there may be faint, sharp-edged beam traces on the anode corresponding to a minor beam system which behaves somewhat differently from the accompanying major beam system. This minor system, often clearly distinguishable from any distortion fringing of the major traces, has its origin in a virtual source formed by those electrons which go directly toward a beam-forming element at which they cannot arrive because of its zero or negative potential. Such

[3] I. Langmuir and K. T. Compton "Electrical discharges in gases," Part II, *Rev. Mod. Phys.*, vol. 3, p. 252; April, (1931).

electrons are diverted at some region relatively close to the element and thus form a weaker trace pattern which behaves differently from the major traces. This minor pattern disappears abruptly at some low positive potential on the beam-forming element, indicating that the electrons in question are absorbed by that element. Some structures of the type of Fig. 1 show this minor system clearly. The current in such a minor system is usually a small percentage of that in the major system. Suitably placed screen grids and other electrodes at positive potentials may, for practical purposes, be sufficiently nearly "currentless" without resort to suppression of the minor beam system. Where the beam-forming electrode is an integral part of the cathode base structure, the minor beam system may be entirely absent. Such an arrangement constitutes a means for avoiding these undesired beam formations.

(6) Another effect which can be misleading in the investigation of beam behavior is the appearance on an anode of a general diffused luminescence which is characterized by the absence of sharp boundaries. The conclusion which has sometimes been drawn that this diffused bombardment is evidence of inefficient beam formation is erroneous in many cases. Such diffused glow is usually present and is caused by high velocity secondary electrons originating at the major traces and returning to the same electrode with sufficient velocity to cause luminescence. The conclusive evidence in certain cases is that, when a beam of cross section determined by the major trace on an anode is let through a slightly larger opening in a similar anode and collected by a more remote electrode at higher potential, there is no *diffused* luminescence on the apertured first anode and its current is very small. Moreover, in the development of devices having positively operated low-current screen and control electrodes the suppression of secondary emission from the anode by slotting or by coating it with soot from a flame results in a large reduction in the residual current to those electrodes even when they are from twenty to forty volts below anode potential. Other observations on specially made devices have confirmed the above interpretation.

(7) Insulator surfaces close to the electron stream act as additional electrodes. Observations by means of beam traces show very clearly the effects of adjacent mica, glass, or other insulators. The effects of such surfaces are consistent with those of so-called "floating" conductors similarly placed. If an insulator surface has, under particular conditions, the ability to emit secondaries in excess of the impinging primaries then it can be made to assume either of two stable "floating" potentials. One of these is close to cathode potential, the other is some

positive potential usually less than that of the highest potential electrode in the device. A transient rise of potential of the insulator can be communicated to it inductively so as to shift it abruptly from its lower to its higher stable value. A sudden change of beam-trace pattern results. It was by means of such observations that a certain erratic behavior of developmental forms of the RCA-58 and other tubes was found to be associated with the polarization of the glass envelope at

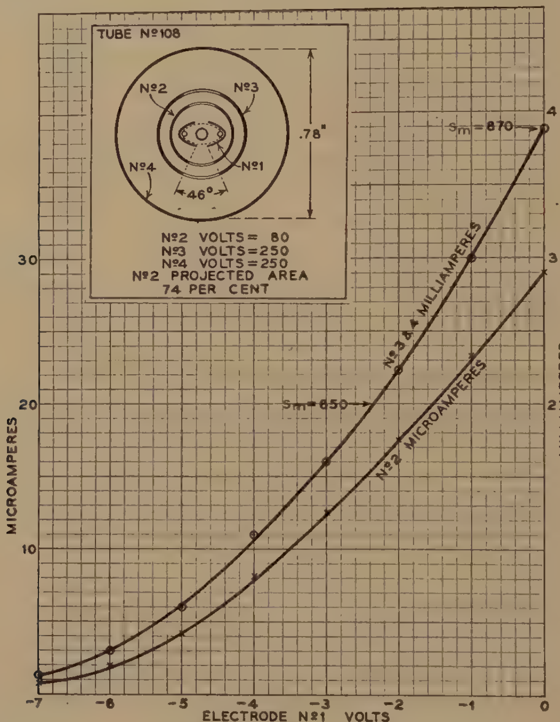

Fig. 11—Low screen current in a two-beam structure.

the higher of the two "floating" potentials. Reduction below the one-to-one value of the secondary emissivity of the interior surface of the glass by coating it with finely divided material such as carbon eliminated the existence of the upper floating potential and the behavior associated therewith.

V. Devices Utilizing Electron Beams

(*1*) *Low-current positive potential electrodes.*

In Fig. 11 is shown a two-beam device employing a beam-forming and space-current controlling electrode No. 1 according to Fig. 10.

The No. 2 electrode may be regarded as the equivalent of a screen grid. For the data shown, the No. 3 electrode is combined with the No. 4 cylinder as an anode. The cathode is of the RCA-57 size, but with a coated length seventy-five per cent of normal. With this in mind, it is apparent that the transconductance of 870 micromhos shown is only moderately lower than normal while the space current is segregated into two beams so that the No. 2, or screen, electrode at 150 volts receives only 0.7 per cent of the space current of four milliamperes at a No. 1 bias of zero. This occurs although the projected area of No. 2 electrode is seventy-four per cent.

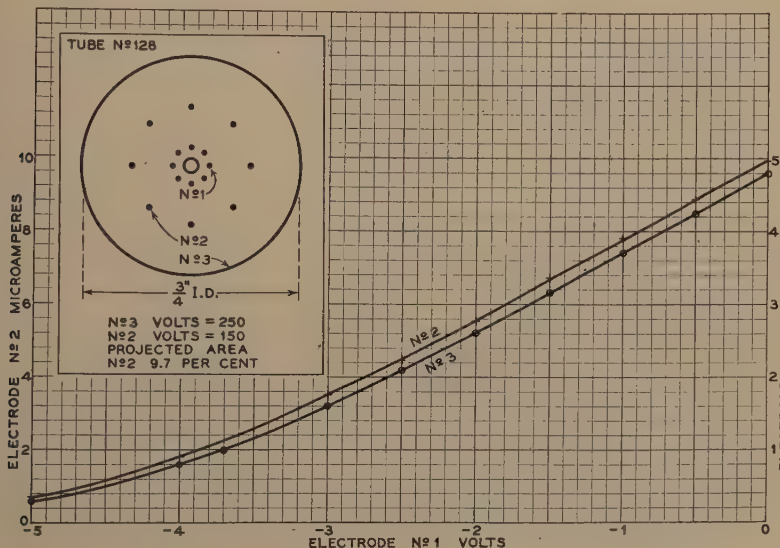

Fig. 12—Anode and screen current in an eight-beam structure.

In the device of Fig. 12, an eight-wire No. 1 electrode has its elements in register with the eight wires of the No. 2 or screen electrode. At the voltages shown the No. 2 electrode receives only 0.2 per cent of the space current at five milliamperes although its projected area is 9.7 per cent. A grid-plate transconductance of 1100 micromhos is observed at five milliamperes.

The device of Fig. 13 has the wires of its No. 2 electrode 2.67 times as large as those in the preceding device but is otherwise the same. One curve shows the anode current as a function of the potential of the No. 2 electrode over a positive range. Such an electrode is thus an example of a low-current positively operated control electrode but its transconductance is extremely poor. Considered as a screen electrode at 150 volts, it receives 0.7 per cent of the anode current although its

projected area is 25.9 per cent. The increase of current to electrode No. 2 above 150 volts is due almost entirely to secondary electrons from the anode. The low current to No. 2 below 150 volts might be thought to be due to loss of secondary electrons from it. That part of the characteristic (not shown) shows very little indication of the flattening observed where secondary emission is considerable. There is ample other evidence that segregation and not secondary emission is the cause of the low current to No. 2.

In the device of Fig. 14, a four-wire longitudinal No. 1 electrode has its openings in register with the four slits of a cylindrical screen or No.

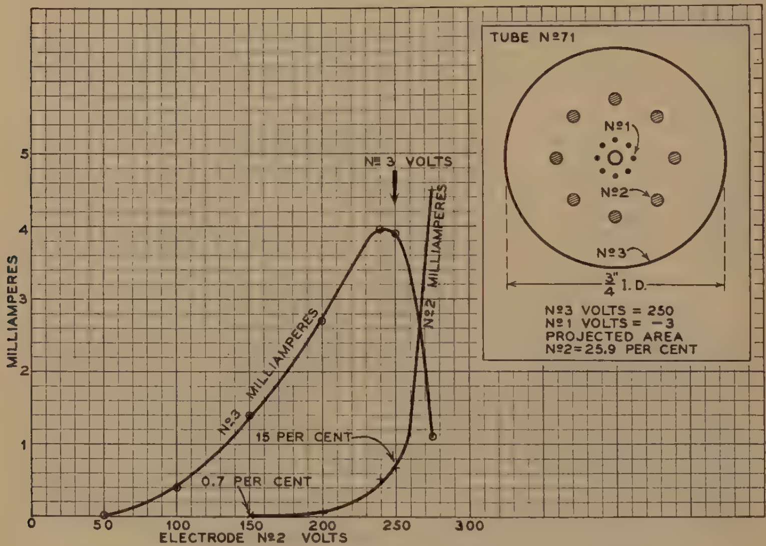

Fig. 13—Low screen current in an eight-beam structure.

2 electrode with a projected area of fifty-nine per cent. The No. 3 and No. 4 electrodes are used together as an anode for the data shown and their peculiar form has no significance for this discussion. Current to the No. 2 electrode at 100 volts positive is about one per cent of the space current when the latter is ten milliamperes.

In the device of Fig. 15, a cylindrical cathode has its emitting area confined to a helical band flush with the equal intervening nonemitting area. The helical emitting band is in register with the openings of a helical wire grid or No. 1 electrode. The helical nonemitting area of the cathode is essentially a beam-forming electrode combined with the cathode and has the effect of confining the beams from the emitting area to the spaces between grid wires. In addition to this nonemitting portion of the cathode two uncoated wires are placed along the cathode

in register with the two support wires of the grid in order to shield them from electron reception. Both No. 1 and No. 2 electrodes are sooted to suppress secondary emission. Grid and anode currents are shown for a positive range of grid potentials. At a grid potential equal to anode potential, only 0.7 per cent of the space current is taken by the grid although its projected area is 10.7 per cent. It will be readily accepted

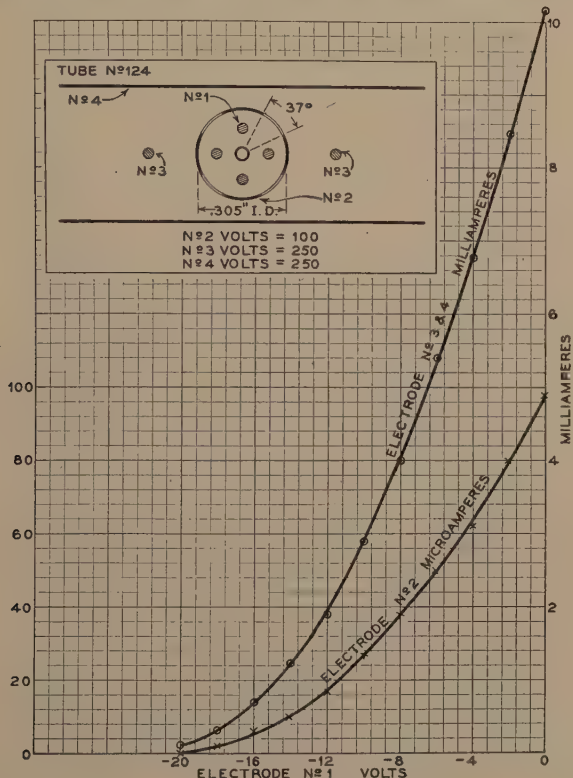

Fig. 14—Low screen current in a four-beam structure.

that the grid at three hundred volts, or fifty volts above the anode potential, does not lose an appreciable percentage of its current by means of secondary emission. Nevertheless the grid there receives only 1.4 per cent of the space current. This constitutes convincing evidence that there is efficient segregation of the space current into beams in the device. It is to be noted that the space current thus segregated and utilized is considerable, being 180 milliamperes to an anode one inch long and three quarters of an inch in diameter, at 250 volts, with the grid at 300 volts. Structures with similar properties have been

made in which the grid elements are all parallel to the cathode axis and in register with corresponding nonemitting strips on the cathode. Such devices can be used as amplifiers of classes B and C with diminished input loss and distortion. In cases where secondary emission from the anode can be effectively suppressed, a new range of operability is made possible, that in which the positive peak voltage of the grid oscillation is greatly in excess of the simultaneous anode potential.

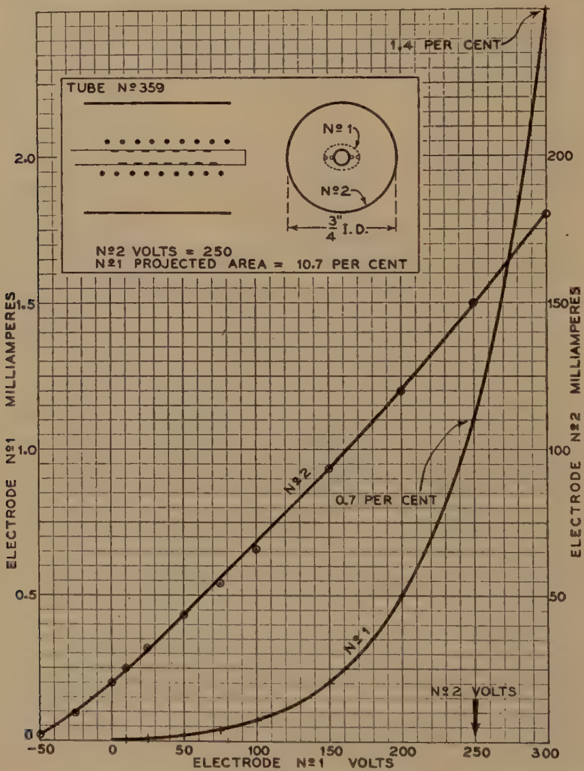

Fig. 15—Low current to a control grid operating at positive potentials.

(2) Special mutual characteristics by means of varying beam width. In the device of Fig. 16, the beam-forming and controlling electrode No. 1 is of the two-wire longitudinal type. The electrode No. 2 has two slits registering with the openings of No. 1. The anode consists of Nos. 3 and 4 in combination and their peculiar form may be disregarded for the purposes of this figure. The graph shows that, for large negative bias of No. 1, the two beams formed by it pass through the slits of No. 2 without contributing appreciable current to the latter. As the bias on No. 1 is decreased, current begins to flow to No. 2 when the beams

become too wide to pass through the slits, whereupon the increment of space current with decreasing No. 1 bias is transferred entirely from the anode combination to the No. 2 electrode as is clearly shown. This result is consistent with the constancy of the ratio of current to beam width shown in Fig. 9. Such a device is typical of a class of devices in which variation of beam width is used to redistribute the space current among positive electrodes in a manner which can be predetermined so

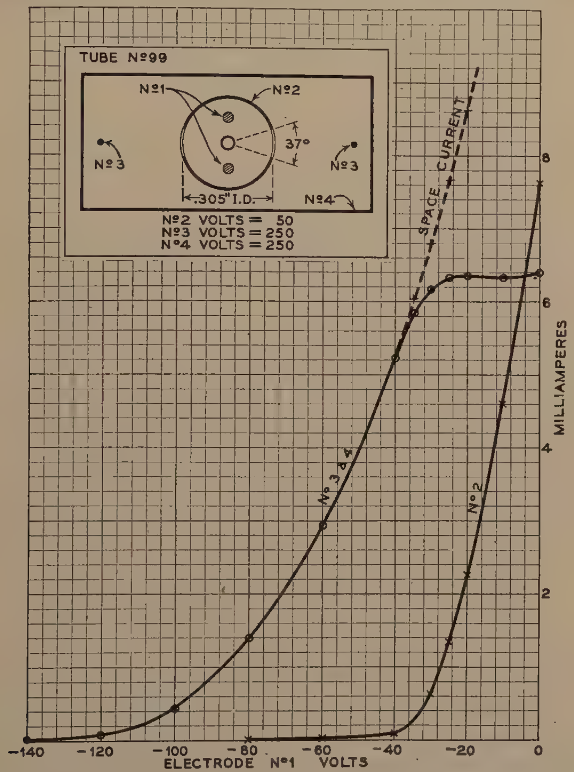

Fig. 16—Special mutual characteristic by varying beam width.

as to give those electrodes mutual and impedance characteristics of various kinds desirable for special purposes. Such characteristics are, in many cases, quite impossible to attain with devices depending on space-charge properties alone. In the above device, for instance, the current to Nos. 3 and 4 shows an upper limit equivalent in effect to emission saturation but not subject to the impracticality of the latter condition. It also illustrates the sharpening of the cutoff on the No. 2 characteristic which results from the rejection onto Nos. 3 and 4 of the smaller values of space current while No. 1 is very negative. Devices

similar in principle but with larger numbers of No. 1 elements have been made and found to operate in similar fashion but with higher transconductance. The electrode No. 2 has been replaced by one consisting of slats placed normally to the beam paths and occupying the positions of the slits in the device shown. Thereby the functions of the No. 2 electrode and the anode are inverted. Suitable shaping of the boundaries of the openings in the No. 2 electrode in that case can be made to alter the relation of anode current to No. 1 voltage, making it either linear or some other predetermined form.

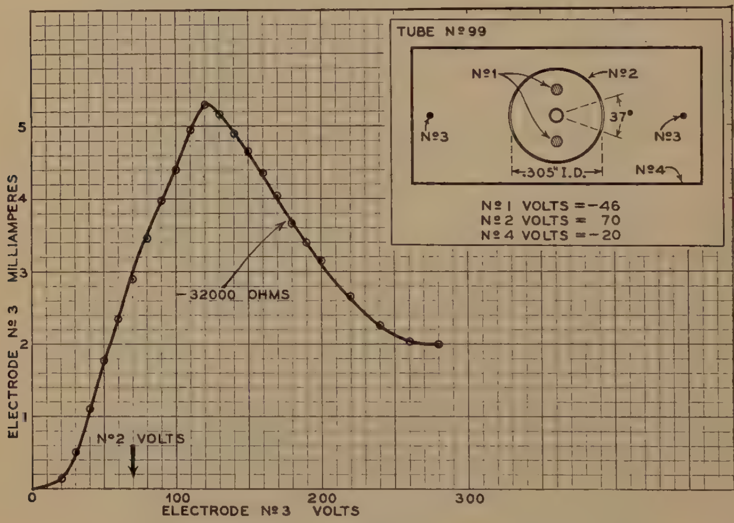

Fig. 17—Negative impedance by change of focus.

(3) *Special impedance characteristics—Negative impedance devices.* The device of Fig. 17 is identical with that of the preceding figure, but is used differently as indicated. The volt-ampere characteristic of electrode No. 3 is shown with the other potentials as there stated. No. 3 consists of two wires of 0.020-inch diameter placed parallel to the cathode and in register with the slits of No. 2. For the potentials shown, the field outside of No. 2 is decelerating and focusing in its effect upon the beams emerging therefrom. A beam is focused before reaching a No. 3 wire when the latter is at low positive potential and the electrons passing through this focus return to No. 2 by orbital paths and form thereon traces readily seen on each side of the slit. As No. 3 potential rises, the focus of a beam approaches and finally arrives on No. 3, when its current becomes the maximum value shown by the graph. The further rise of No. 3 potential gives the falling or negative imped-

ance portion of the characteristic as shown. This decrease of current to No. 3 is caused by the recession of the beam focus to a region between Nos. 3 and 4 and the consequent return of most of the electrons to No. 2. Thus, the negative impedance of No. 3 may be said to be due to a process of defocusing with respect to that electrode. A variety of devices are operable upon this principle. It is noteworthy that the negative slope occurs while the potential of No. 3 is higher than that of any other electrode in the device, thus entirely excluding the possibility that secondary emission from No. 3 is responsible for its negative impedance.

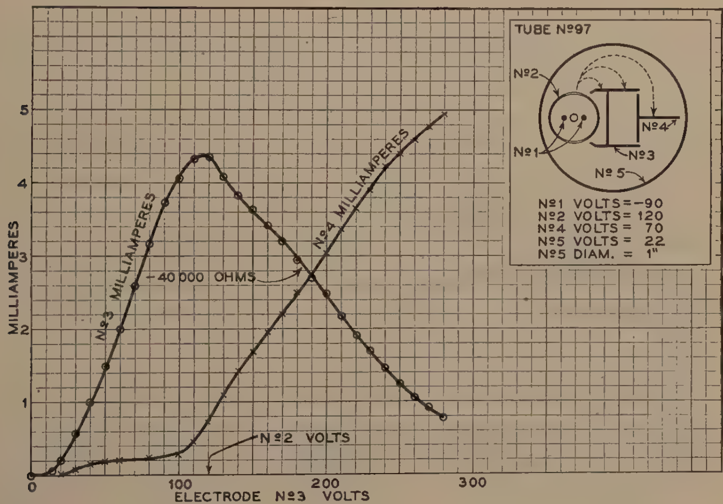

Fig. 18—Negative impedance by change of electron trajectories.

In the device of Fig. 18, the two-beam electron "gun" projects its beams tangentially with respect to the equipotential surfaces of a roughly radial electric field existing between Nos. 3 and 5. The latter is shown at a positive potential of twenty-two volts but has also been used at zero and negative potentials. No. 4 may have any positive potential in a wide range. As the potential of No. 3 is increased, the electron trajectories increase in length as indicated by the successively longer dotted lines shown on the drawing of the device. Eventually, the beams miss No. 3 entirely and are received on No. 4. During the transition of the beams from No. 3 to No. 4, the former has a negative impedance as shown. As in the preceding device, such negative impedance cannot be caused by secondary emission. This device was rendered slightly gassy and the beam paths were clearly seen to behave as stated.

Another negative impedance device, not shown, has been made in which the property stated under Section IV, Subsection (4), is utilized. The structure consists of, in order: a cathode, a beam-forming electrode used at a positive potential, a registering slit electrode used at a higher positive potential, and an anode or collector used at a still higher potential. At some potential on the slit electrode, each beam passes through its corresponding slit and the current to the electrode is small. As the potential of the slit electrode is decreased, each beam becomes too wide to pass through the corresponding slit; hence the current to the electrode increases and a negative impedance results.

The last three devices are examples of a class of devices in which the negative impedance of an electrode is due to its direct electrostatic effect upon the electron paths in such a way as to divert current from itself as its potential increases. No co-operating changes of potential on other electrodes are necessary and the device is functionally equivalent to a secondary-emission dynatron and has the advantage that the constancy of its negative impedance is dependent on geometrical constancy of the structure and not upon the sometimes unreliable secondary emissivity. Such devices have been made to oscillate at audio frequencies and at fifty megacylces. The starting and stopping of oscillation are accompanied by visible changes in the trace patterns.

(*4*) *Luminous trace voltmeters.*

The width of a beam trace, or of the dark space between traces, is a measure of the potential of the beam-forming electrode relative to the potentials of other electrodes in the device. This property is illustrated in Figs. 1 and 8. It has found application in tube voltmeters in which the "pointer" is the edge of a luminous trace, and the scale is the surface of the electrode carrying the trace. Such a device, the RCA 6E5, is now used commercially as a tuning indicator and for other purposes. The convenient form of the structure in this case is largely due to H. M. Wagner. The beam-forming electrode is here actuated by the output of a triode amplifier included in the tube.

VI. Conclusion

A new array of properties of electron discharge devices is introduced by systematically utilizing beam formation. The combination of these new properties with those already in use furnishes an augmented supply of means for the improvement of existing types of devices, and for the creation of devices having entirely new properties to fulfill the new requirements which inevitably arise.

Experience with such structures as represented here indicates that

the utilization of beam formations is consistent with economy of cathode power and with the attainment of conductances and transconductances of magnitudes within the useful range even at low voltages. The difficulties of design of such structures are considerable in the present stage of the art. The difficulties of construction are, in many cases, not extraordinary.

Acknowledgment

The writer wishes to express his appreciation of the sponsorship of this work by Mr. B. J. Thompson, and also of the valuable co-operation of Mr. H. M. Wagner in the later stages of it.

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TRANSMISSION OF ELECTROMAGNETIC WAVES IN HOLLOW TUBES OF METAL*

By

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Summary—*Electromagnetic energy may be transmitted through the inside of hollow tubes of metal, provided the frequency is greater than a certain critical value; this value is inversely proportional to the tube radius and to the dielectric coefficient for the tube interior. Calculations and measurements of the more important characteristics of this new kind of transmission system have been made, and the conditions for minimum attenuation obtained. Terminal devices for connecting a hollow pipe system to a biconductor system and others, in the form of horns, for directly radiating radio waves, have been developed; these electromagnetic horns may also be fed with ordinary coaxial lines. Certain types of terminals act as sharply resonant hollow tube elements. Several independent communication channels may be established within a single pipe line by utilizing distinct types of waves for each channel in a unique kind of multiplex operation. A section of a hollow tube may be used as a high-pass filter. Although presupposing adequate technique for the generation and utilization of the shortest radio waves, this new system possesses several features, among which are a minimum dielectric loss, substantially perfect shielding, and a simplicity of structure.*

Introduction

THIS paper deals with the transmission of electromagnetic energy at high frequency through the inside of hollow tubes of metal. Although single conductor and multiple conductor transmission circuits of wire, cable, and concentric tube form have been employed for guiding radio-frequency energy from one point to another, the interior of a hollow conducting cylinder does not appear to have been previously suggested or used in this capacity.[1]

The literature is singularly sparse on the subject of waves within

* Decimal classification: R110. Original manuscript received by the Institute, May 15, 1936. Presented before joint U.R.S.I.-I.R.E. meeting, Washington, D. C. May 1, 1936, and before Boston Section of the I.R.E., May 22, 1936.

[1] Some time after this paper was scheduled for presentation before this U.R.S.I.-I.R.E. meeting, the author received a communication from Dr. G. C. Southworth of the Bell Telephone Laboratories, stating that they had also been working on this problem and had two papers in preparation which have since appeared in the *Bell System Technical Journal*. These papers are entitled "Hyper-Frequency Wave Guides" and are by G. C. Southworth[2] (General Considerations and Experimental Results) and J. R. Carson, Sallie P. Mead, and S. A. Schelkunoff[3] (Mathematical Theory).

[2] G. C. Southworth, *Bell Sys. Tech. Jour.*, vol. 15, pp. 284–309; April, (1936).

[3] J. R. Carson, Sallie P. Mead, and S. A. Schelkunoff, *Bell Sys. Tech. Jour.*, vol. 15, pp. 310–333; April, (1936).

conducting cylinders. Heaviside,[4] Thompson,[5] and Rayleigh[6] gave brief discussions of limited aspects of the problem. Indeed, Heaviside indicated at one point[7] that waves could not be sent through a uniform tube without a second center conductor. Rayleigh's paper (1897), although confined to perfectly conducting tubes, differentiates between the various possible types of waves and obtains the critical frequencies below which they cannot exist. Although somewhat related, the work of Hondros and Debye,[8] of Zahn,[9] and of Schriever[10] on waves on dielectric wires applies to a different physical situation. The recent paper by Schelkunoff[11] is confined to problems associated with coaxial conductors. The hollow conducting cylinder, apparently lost sight of since Rayleigh's original paper, reappeared in the literature in a paper by Bergmann and Kruegel[12] (1934), who measured both the field inside a short hollow metal cylinder and the radiation from its open end when a half-wave coaxial antenna was properly excited; the use of hollow pipes for the conduction of ultra-high-frequency energy was not suggested. The transmission characteristics and the attenuation in finitely conducting pipes as well as means for exciting and for receiving the waves at the ends have not been previously discussed.*

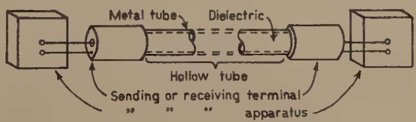

Fig. 1—Elements of a hollow tube transmission system.

A hollow tube transmission system (see Fig. 1) would comprise

(1) A single metal conductor in the form of a tube or pipe having appropriate dimensions relative to the frequency of the excitation and having its interior filled with dielectric material in gaseous, liquid, or

[4] Oliver Heaviside, "Electrical Papers," (1892), Reprint, Boston, (1925), vol. I, pp. 443–467.

[5] J. J. Thomson, "Recent Researches in Electricity and Magnetism," Oxford, (1893), p. 344.

[6] Lord Rayleigh, "Scientific Papers," (1897), vol. IV, pp. 227–280.

[7] Oliver Heaviside, "Electromagnetic Theory," Electrician Pr. and Pub. Co., London, (1893), vol. I, p. 399 *et seq.*

[8] D. Hondros and P. Debye, *Ann. der Phys.*, vol. 32, p. 465, (1910).

[9] H. Zahn, *Ann. der Phys.*, vol. 49, pp. 907–933, (1916).

[10] O. Schriever, *Ann. der Phys.*, vol. 63, pp. 645–673, (1920).

[11] S. A. Schelkunoff, *Bell. Sys. Tech. Jour.*, vol. 13, p. 533, (1934).

[12] L. Bergmann and L. Kruegel, *Ann. der Phys.*, vol. 21, pp. 113–138; October, (1934).

* Note added in proof. Mr. E. F. Northrup has written the author that he, assisted by Mr. W. Wrighton, carried on some experiments in February, 1908, on transmitting radiation through cardboard tubes covered with tin foil. These early unpublished experiments were abandoned after several weeks' work.

solid form, or evacuated, connecting the sending and the receiving points:

(2) A terminal device at each end of the tube for connecting the uniconductor hollow tube proper to the biconductor sending and receiving apparatus for supplying modulated high-frequency energy at one or both ends and for receiving and demodulating the energy transmitted to the opposite end: or

(3) One terminal device as in (2) at one end and a device at the other end for radiating the transmitted energy into the atmosphere directly.

The transmission circuit is formed by the dielectric material or vacuum filling the tube interior and the inner surface of the conducting tube. The tube acts not only as the conductor for the transmission system but also as a highly effective shield against interference, because, at the high frequencies to be used, the skin effect confines the circuit substantially to the interior of the tube. Operation with currents whose free-space wave length is much less than one meter is necessary if the tube size is to remain reasonably small. Hence, the commercial use of hollow tube systems, which presumes adequate technique for generation, amplification, etc., of the shortest radio waves, will probably not be an immediate development. Nevertheless, this type of high-frequency circuit possesses several desirable features, such as the following: a minimum of dielectric loss, substantially perfect shielding, presumable low cost of construction and installation, and possibility of rugged mechanical design. It would seem that a hollow tube might be the ideal "conductor" for waves of several centimeters length and less, as naturally suited to this region of the spectrum as a pair of copper wires is to the lower frequency region. In view of the rapid development at this time of ultra-high-frequency technique for telephone, telegraph, and television purposes, an investigation of the possibilities and limitations of the hollow tube transmission system is of immediate interest. This paper presents some of the results of a theoretical and experimental study carried on by the author for some years. The electromagnetic and the circuit characteristics of the hollow tube proper are first presented; this part of the system may be analyzed rigorously because of its simple geometrical configuration. Terminal devices are then discussed qualitatively; this part of the system has not yielded to mathematical analysis. Finally, the results of an experimental investigation are presented in which the major aspects of the theory are substantiated.

Equations for the Hollow Tube Transmission System

The geometrical configuration is shown in Fig. 2 (a) where an infinitely long hollow cylinder is assumed to be cut out of a homogeneous conductor of otherwise infinite extent. The space inside of the cylinder is assumed to be a nonconductor of dielectric constant ϵ_1 and permeability μ_1; it might be vacuum, air, or some other gas at a suitable pressure, a liquid, or an appropriate solid material. The conductor is assumed to have a conductivity σ_2 and a permeability μ_2. The current penetration into metallic conductors at the frequencies of interest in this problem is so small that the equations developed below apply with accuracy to metal tubes of ordinary thickness of wall. Analytically, it

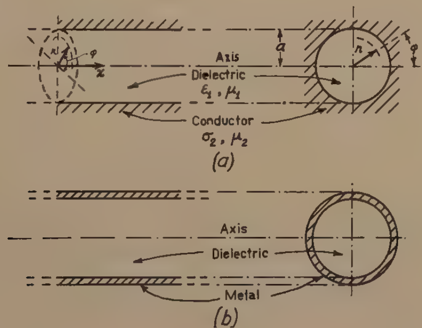

Fig. 2—(a) Polar co-ordinates for cylindrical cavity in conducting medium of infinite extent.
(b) Configuration of actual electromagnetic pipe.

is more convenient to assume that the conductor is unbounded externally; actually, the transmission system under consideration is bounded and is a metal tube or pipe such as that shown in Fig. 2 (b).

A practical system of units will be used throughout this paper, wherein:

E = electric field intensity in volts per centimeter

H = magnetic field intensity in amperes per centimeter

σ = conductivity in mhos per centimeter

μ = permeability in henrys per centimeter
(For air, $\mu = \mu_0 = 4\pi \cdot 10^{-9}$)

ϵ = dielectric constant in farads per centimeter
(For air, $\epsilon = \epsilon_0 = 10^{-11}/36\pi$)

$c = 1/\sqrt{\mu_0 \epsilon_0}$ = velocity of light $= 3 \times 10^{10}$ centimeters per second.

The Maxwell equations in this notation are

Differential form	Integral form	
$\operatorname{curl} H = \sigma E + \dfrac{\partial \epsilon E}{\partial t}$	$\oint H \cdot ds = \int \left(\sigma E + \dfrac{\partial \epsilon E}{\partial t} \right) \cdot da$	(1)
$\operatorname{curl} E = -\dfrac{\partial \mu H}{\partial t}$	$\oint E \cdot ds = -\dfrac{d}{dt} \int \mu H \cdot da$	(2)

Our interest lies in the propagation of waves with a simple harmonic time variation of angular frequency $\omega = 2\pi f$ radians per second down the inside of the tube in the positive direction. We therefore introduce the exponential factor $e^{-hx+i\omega t}$ into the field vectors E and H in the following manner:

We put

$$E = Ee^{-hx+i\omega t}$$
$$H = He^{-hx+i\omega t} \tag{3}$$

and let the new E and H be independent of both x and t; they may be functions of r and ϕ. The quantity h is the propagation constant. In general, it has a real part α, the attenuation constant, and an imaginary part, β, the phase constant. The determination of $h = \alpha + i\beta$ is one of the principle undertakings of this investigation. In order to have (3) represent a wave traveling in the $+x$ direction, we must always take the root of h with a positive real part. If we substitute (3) into (1) and (2) and transform to the cylindrical co-ordinates of Fig. 2(a), the resulting six equations may be divided into two groups, one of which corresponds to circular currents around the tube and the other to longitudinal currents along it. The latter group alone[13] applies to our problem where the field will have the general properties,

$$E = E_x, E_r \qquad H = H_\phi$$

$$E_\phi = H_x = H_r = \frac{\partial}{\partial \phi} = 0.$$

[13] The analysis presented here applies specifically to the lowest mode of oscillation obtainable when the field is describable by E_x, E_r, and H_ϕ, and this type of wave will be referred to as "longitudinal." The experimental work, however, includes the case of the lowest frequency mode appropriate to the set H_x, H_r, and E_ϕ, and this wave type will be called "transverse." The higher modes of oscillation, of which there are a great number, have not been investigated by the author. The following analysis is based on the method developed by A. Sommerfeld[14] for the related problem of waves on wires.

[14] A. Sommerfeld in Riemann-Weber, "Differential- und Integralgleichungen," F. Vieweg & Sohn, Braunschweig, vol. II, 7th edition, (1927).

This group of three equations is

$$E_r = \frac{h}{\sigma + i\omega\epsilon} H_\phi \tag{4a}$$

$$hE_r + \frac{dE_x}{dr} = i\omega\mu H_\phi \tag{4b}$$

$$\frac{drH_\phi}{dr} = (\sigma + i\omega\epsilon) r E_x. \tag{4c}$$

Substituting (4a) into (4b) and the result into (4c) gives the expressions for $H\phi$ and E_r in terms of E_x and the differential equation for E_x; viz.,

$$H_\phi = -\frac{\sigma + i\omega\epsilon}{k^2 + h^2} \frac{dE_x}{dr} \tag{5}$$

$$E_r = -\frac{h}{k^2 + h^2} \frac{dE_x}{dr} \tag{6}$$

$$\frac{d^2 E_x}{dr^2} + \frac{1}{r} \frac{dE_x}{dr} + (k^2 + h^2) E_x = 0. \tag{7}$$

The abbreviation

$$k^2 = \omega^2 \mu\epsilon - i\omega\mu\sigma \tag{8}$$

was introduced in the above expressions. The root of k having a positive imaginary part and a negative real part must always be used in these calculations where the wave is propagated in the positive x direction. The differential equation (7) is solved by the Bessel and the Hankel functions[15]

$$J_0(r\sqrt{k^2 + h^2}), \quad H_0{}^1(r\sqrt{k^2 + h^2}), \quad H_0{}^2(r\sqrt{k^2 + h^2}).$$

Proper choice of these solutions will give the field inside of the hollow tube and the field within the metal conductor.

The two functions $H_0{}^1$ and $H_0{}^2$ become infinite at the center of the hollow tube ($r=0$) and hence J_0, which is finite for $r=0$, must be used to represent the field within the dielectric, denoted by the subscript 1. Adopting the notation

$$k_1{}^2 = \omega^2 \mu_1 \epsilon_1, \quad k_1 = \omega\sqrt{\mu_1 \epsilon_1} \tag{9}$$

[15] Jahnke-Emde, "Tables of Functions," Teubner, Leipzig, (1933).

for the value of k within the dielectric and using the relation

$$\frac{dE_x}{dr} = -C\sqrt{k_1^2 + h^2}\, J_1(r\sqrt{k_1^2 + h^2}),$$

we obtain the following expressions for the field inside the tube:

$$\text{In the dielectric} \begin{cases} E_x = CJ_0(r\sqrt{k_1^2 + h^2}) & (10a) \\ E_r = C\dfrac{h}{\sqrt{k_1^2 + h^2}} J_1(r\sqrt{k_1^2 + h^2}) & (10b) \\ H_\phi = C\dfrac{i\omega\epsilon_1}{\sqrt{k_1^2 + h^2}} J_1(r\sqrt{k_1^2 + h^2}) & (10c) \end{cases}$$

where C is a constant determined by the strength of the excitation.

In the conducting wall of the tube $(r>a)$ the solution H_0^1 must be used to represent the field, because H_0^2 becomes infinite at $r=\infty$ and J_0 vanishes for $r\rightarrow\infty$ only as $1/\sqrt{r}$, which is not rapid enough to satisfy energy conditions at infinity. A proof of this statement is given by Sommerfeld.[14] Dropping the superscript 1 of H_0^1 and denoting the value of k^2 for the metal by k_2^2,

$$k_2^2 = \omega^2\mu_2\epsilon_2 - i\omega\mu_2\sigma_2 \cong -i\omega\mu_2\sigma_2$$

$$k_2 = (-1 + i)\sqrt{\frac{\omega\mu_2\sigma_2}{2}} \qquad (11)$$

we obtain the following expressions for the field in the conducting wall of the tube:

$$\text{In the conductor} \begin{cases} E_x = AH_0(r\sqrt{k_2^2 + h^2}) & (12a) \\ E_r = A\dfrac{h}{\sqrt{k_2^2 + h^2}} H_1(r\sqrt{k_2^2 + h^2}) & (12b) \\ H_\phi = A\dfrac{\sigma_2}{\sqrt{k_2^2 + h^2}} H_1(r\sqrt{k_2^2 + h^2}) & (12c) \end{cases}$$

where A is another constant determined by the strength of the excitation.

The quantities h and A/C are as yet undetermined. We may determine h by applying the boundary conditions of our problem to the field expressions (10) and (12); the value of A/C is then determined from these equations. The boundary conditions require the tangential components of the electric field and of the magentic field to be continuous at the tube surface $r=a$; i.e.,

$$E_{z1}(a) = E_{z2}(a), \quad H_{\phi 1}(a) = H_{\phi 2}(a). \tag{13}$$

Substituting in (13) the values from (10) and (12) gives the two equations

$$CJ_0(a\sqrt{k_1^2 + h^2}) = AH_0(a\sqrt{k_2^2 + h^2}) \tag{14a}$$

$$C\frac{i\omega\epsilon_1}{\sqrt{k_1^2 + h^2}} J_1(a\sqrt{k_1^2 + h^2}) = A\frac{\sigma_2}{\sqrt{k_2^2 + h^2}} H_1(a\sqrt{k_2^2 + h^2}). \tag{14b}$$

Dividing (14a) by (14b) and rearranging, we obtain the transcendental equation

$$\left.\begin{aligned} \frac{i\omega\epsilon_1 z H_0(z)}{\sigma_2 H_1(z)} &= \frac{yJ_0(y)}{J_1(y)} \\ y &= a\sqrt{k_1^2 + h^2} \\ z &= a\sqrt{k_2^2 + h^2} \end{aligned}\right\}. \tag{15}$$

This equation for h cannot be solved generally, but several special cases can be solved; fortunately, they are the most interesting cases from a practical viewpoint. The solutions of (15) will be obtained and the other aspects of the transmission of electromagnetic waves in hollow tubes will be developed in following sections of this paper.

Perfectly Conducting Hollow Tubes

The limiting case in which the conductivity of the tube is assumed to be infinite is approached in certain respects rather closely with metals such as copper and aluminum and it serves to establish the approximate behavior of the system for large finite values of conductivity.

As $\sigma_2 \rightarrow \infty$, k_2 increases without limit, becoming an infinite complex number with a positive imaginary part; therefore, $h^2 \ll k_2^2$, and the left-hand side of (15) approaches the value zero, hence

$$\frac{yJ_0(y)}{J_1(y)} = 0.$$

Consideration of the nature of the zero and the first order Bessel functions shows that the equivalent of this equation is

$$J_0(y) = 0. \tag{16}$$

This same equation may also be obtained by an application of the boundary condition for perfect conductors; viz., the vanishing of the tangential component of the electric field on the surface of the conductor, to the expression (10a), which gives

$$E_z(a) = CJ_0(y) = 0.$$

This equation is identical to (16). The transcendental equation (16) has an infinite number of roots $y_0, y_1, \cdots$. The smallest root $y_0 = 2.4048$ is the one of principal importance in the transmission of energy through the tube, and the waves corresponding to this root will be called the principal waves and this mode of excitation the principal mode. Auxiliary modes and waves associated with $y_1, y_2, \cdots$ will not be treated in this paper.

For the principal waves, the solution of (16) gives

$$a\sqrt{k_1^2 + h^2} = y_0 = 2.4048$$

or,

$$h^2 = \left(\frac{y_0}{a}\right)^2 - k_1^2 \tag{17}$$

$$h = \sqrt{\left(\frac{y_0}{a}\right)^2 - k_1^2}.$$

Only real values of $h^2 < k_1^2$ can satisfy (17), hence it is convenient to write

$$\left.\begin{aligned} h &= i\sqrt{k_1^2 - \left(\frac{y_0}{a}\right)^2} = i\sqrt{\omega^2\mu_1\epsilon_1 - \left(\frac{y_0}{a}\right)^2} \\ &= i\beta \\ \beta &= \sqrt{\omega^2\mu_1\epsilon_1 - \left(\frac{y_0}{a}\right)^2} \quad \text{(real)} \end{aligned}\right\}. \tag{18}$$

Thus we find that for perfectly conducting tubes, $h = 0 + i\beta$ — there is no attenuation and the phase constant is given by (18). When the frequency f of the excitation is too low, β is imaginary and there can be no wave propagation down the tube. When the frequency is high enough, waves are propagated without attenuation at a phase velocity ω/β that varies with the tube radius. Thus, the frequency spectrum for any tube may be divided into a region of *complete attenuation* in which the tube is opaque and a region of *perfect transmission* in which the tube is transparent. The two regions are separated by the critical frequency f_0

$$\left.\begin{aligned} f_0 &= \frac{y_0}{2\pi\sqrt{\mu_1\epsilon_1}} \cdot \frac{1}{a}, \\ f &< f_0 \cdots \text{complete attenuation} \\ &> f_0 \cdots \text{perfect transmission} \end{aligned}\right\}. \tag{19}$$

For frequencies within the region of perfect transmission the following expressions are obtained:

$$\begin{aligned}
\text{Wave length in tube} &= \lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\sqrt{\omega^2\mu_1\epsilon_1 - \left(\frac{y_0}{a}\right)^2}} \\
\text{Phase velocity} &= v_p = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\omega^2\mu_1\epsilon_1 - \left(\frac{y_0}{a}\right)^2}} \qquad (20) \\
\text{Group velocity} &= v_g = \frac{1}{\frac{d\beta}{d\omega}} = \frac{\sqrt{\omega^2\mu_1\epsilon_1 - \left(\frac{y_0}{a}\right)^2}}{\omega\mu_1\epsilon_1}.
\end{aligned}$$

Equations (18), (19), and (20) will now be discussed for an ideal dielectric $\mu_1 = \mu_0$, $\epsilon_1 = \epsilon_0$, the case closely approximated by air and most other gases. It is convenient to substitute in the equations

$$\mu_1\epsilon_1 = \mu_0\epsilon_0 = 1/c^2,$$

whereupon we obtain

$$\begin{array}{l} \text{For air, etc.} \\ \epsilon_1 = \epsilon_0 \\ \mu_1 = \mu_0 \end{array} \left\{ \begin{aligned}
\beta &= \sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{y_0}{a}\right)^2} \\
f_0 &= \frac{c y_0}{2\pi} \cdot \frac{1}{a} \cong 1.148 \times 10^{10} \frac{1}{a} \\
\lambda_0 &= \frac{c}{f_0} \cong 2.615a \\
\lambda &= \frac{2\pi}{\sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{y_0}{a}\right)^2}} \\
v_p &= \frac{\omega}{\sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{y_0}{a}\right)^2}} \\
v_g &= \frac{c^2}{v_p}.
\end{aligned} \right. \qquad (21)$$

The critical frequency f_0 is inversely proportional to the tube radius a, hence the smaller the tube, the larger the frequency of operation.

Numerical values of f_0 are given by the curve of Fig. 3, where the corresponding critical free-space wave length $\lambda_0 = c/f_0$ of the excitation

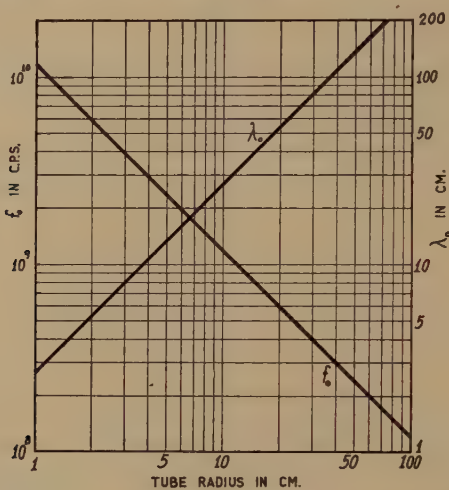

Fig. 3—Critical frequency and critical free-space wave length vs. inner radius of tube.

has also been plotted. The practical application of the hollow tube transmission system, if restricted to tube diameters of 4 to 100 centi-

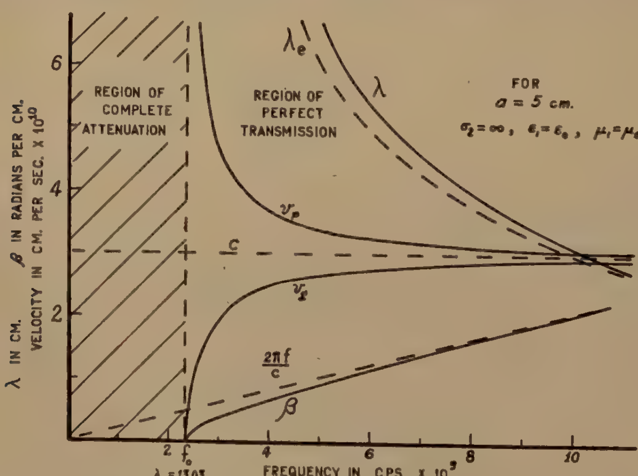

Fig. 4—Illustrating the transmission characteristics of a perfectly conducting pipe.

meters, clearly requires the use of frequencies between 2.3×10^8 and about 6×10^9 cycles; i.e., wave lengths in free space λ_e of the excitation between about 5 and 100 centimeters. For a tube of given radius a,

the values of β, λ, v_p, and v_g all vary with the frequency of excitation over the transmission region. In Fig. 4 curves of these quantities are reproduced for a tube of 5 centimeters radius; however, these curves are typical for tubes of any radius. The phase velocity is greater than the velocity of light c, but it approaches c closely when $\lambda_e < a$. The group velocity is less than c, but it also approaches this value under the above condition. In the limit when $\lambda_e \ll a$, we find $v_p = v_g = c$, $\beta = \omega/c$, and $\lambda = \lambda_e$, and wave propagation occurs much in the same way as it does on a single wire of radius large compared to λ_e or in a half space.[14] Fig. 5 shows curves of β vs. f for several tube radii.

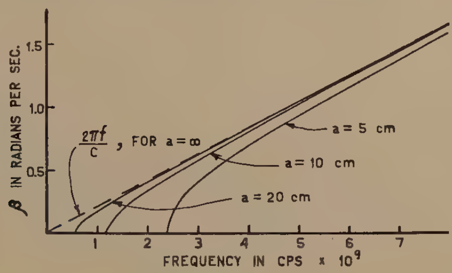

Fig. 5—Shift in the phase function β with radius of the pipe.

Having obtained the value of h^2 (17) we find from (10) that the field inside the tube is given by

$$\begin{array}{l} \text{For air, etc.} \\ \mu_1 = \mu_0 \\ \epsilon_1 = \epsilon_0 \\ \sigma_2 = \infty \end{array} \quad \left\{ \begin{aligned} E_x &= CJ_0\left(y_0 \frac{r}{a}\right) \\ E_r &= C\frac{i\beta a}{y_0} J_1\left(y_0 \frac{r}{a}\right) \\ H_\phi &= C\frac{i\epsilon_0 a}{y_0} J_1\left(y_0 \frac{r}{a}\right). \end{aligned} \right. \tag{22}$$

The radial component E_r and the magnetic field H_ϕ both vanish at the axis of the tube and the axial component E_x vanishes at the tube wall $r = a$. The distribution of these intensities over the cross section of the tube is shown in Fig. 6; this distribution, within the transmission region, is independent of the frequency of excitation and of the tube size.

Before calculating the lines of electric field intensity, we must add the exponential factor according to (3) to E_x and E_r in (22) giving

$$\begin{aligned} E_x &= CJ_0\left(y_0 \frac{r}{a}\right) e^{-i\beta x} \cdot e^{i\omega t} \\ E_r &= C\frac{i\beta a}{y_0} J_1\left(y_0 \frac{r}{a}\right) e^{-i\beta x} \cdot e^{i\omega t}. \end{aligned} \tag{23}$$

A plot of the field at any instant of time may be obtained by putting $t = \text{constant} = 0$, whereupon

$$\begin{aligned} |E_x| &= CJ_0\left(y_0 \frac{r}{a}\right) \cos \beta x \\ |E_r| &= C \frac{\beta a}{y_0} J_1\left(y_0 \frac{r}{a}\right) \sin \beta x, \end{aligned} \tag{24}$$

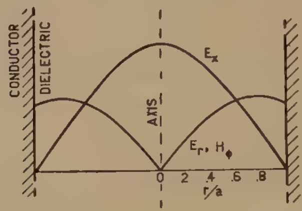

Fig. 6—Distribution of the electric and of the magnetic field components on a diameter of the tube; units and relative magnitudes to arbitrary scale.

and by forming the differential equation for the lines of force

$$\frac{d(\beta x)}{d\left(\frac{r}{a}\right)} = \frac{|E_x|}{|E_r|} = \frac{y_0}{\beta a} \frac{J_0\left(y_0 \frac{r}{a}\right)}{J_1\left(y_0 \frac{r}{a}\right)} \cot \beta x. \tag{25}$$

This equation, which can be solved by graphical methods, leads to the picture given by Fig. 7 of the electric lines of force for the principal

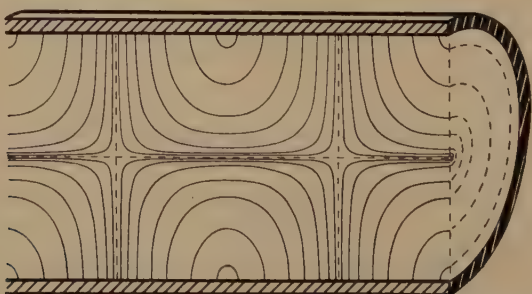

Fig. 7—Plot of the lines of electric intensity in a plane through the axis for a hollow tube wave of the longitudinal type. A sinusoidal time variation is assumed and the plot is for an arbitrary instant of time.

wave in a perfectly conducting tube. This figure shows a cross section through the axis of the tube and one at right angles thereto. The wave propagation may be conceived to be a movement down the tube of this field structure with the velocity $v_p = \omega/\sqrt{(\omega/c)^2 - (y_0/a)^2}$ and without any alteration of shape or of magnitude.

In the engineering treatment of transmission lines, the characteristic impedance is generally an indispensable quantity. The absence of a second parallel metal conductor makes this conception somewhat artificial in the hollow tube system. The displacement current inside the tube in a sense assumes the rôle of the return conductor, and therefore we may define a quantity Z_0, which we will also call the characteristic impedance, relating the strength of the longitudinal current along the tube to the transverse electromotive force between tube axis and wall. The transverse electromotive force in volts is given by

$$V = \int_0^a E_r dr = C\frac{i\beta a}{y_0}\int_0^a J_1\left(y_0\frac{r}{a}\right)dr = C\frac{i\beta a^2}{y_0{}^2}. \tag{26}$$

The longitudinal current in amperes may be computed from the Maxwell equation (1) in integral form

$$I = \oint_{r=a} H_\phi d\phi = 2\pi a H_\phi(a) = C\frac{i\omega\epsilon_0 a^2 2\pi}{y_0}J_1(y_0). \tag{27}$$

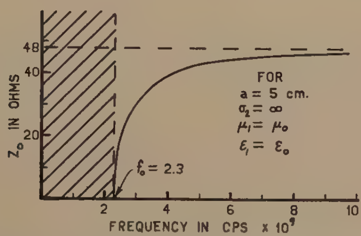

Fig. 8—Characteristic impedance vs. frequency for a five-centimeter radius perfectly conducting air-filled tube.

We define the characteristic impedance Z_0 in ohms as

$$Z_0 = \frac{V}{I} = \frac{\beta/\omega}{2\pi y_0\epsilon_0 J_1(y_0)} = 1.44 \times 10^{12}\frac{\beta}{\omega} = \frac{1.44 \times 10^{12}}{v_p}. \tag{28}$$

As the frequency increases without limit we get the asymptotic value for Z_0

$$\lim_{f\to\infty} Z_0 = \frac{1.44 \times 10^{12}}{c} = 48 \text{ ohms}.$$

As seen from (28) and from Fig. 4, Z_0 is zero at the critical frequency. A curve of Z_0 vs. f for a tube of five centimeters radius is reproduced in Fig. 8; similar results may be obtained for tubes of other radii. The hollow tube is essentially a low impendance transmission system when considered on the basis of Z_0 defined as in (28).

Imperfectly Conducting Hollow Tubes

If the conductivity of the tube is finite but very large, as in the case with copper, aluminum, etc., we may secure an approximate solution for h from the transcendental equation (15) by replacing the two Hankel functions with their asymptotic expressions for large arguments and by neglecting $h^2 \ll k_2^2$. In this manner we get

$$H_0\,(ak_2)/H_1(ak_2) = i,$$

and it follows that

$$\left.\begin{aligned} u &= \frac{yJ_0(y)}{J_1(y)}, \\ u &= -\frac{\omega\epsilon_1 a k_2}{\sigma_2} \end{aligned}\right\} \tag{29}$$

is the transcendental equation for h. The conductivity being very high, y will not depart much from its value y_0 for infinite conductivity and consequently we may assume

$$y = y_0 + \Delta \tag{30}$$

where Δ is a small complex quantity to be determined. Under this hypothesis (29) becomes

$$uJ_1(y_0 + \Delta) = (y_0 + \Delta)J_0(y_0 + \Delta). \tag{31}$$

We may neglect Δ everywhere in this equation except in $J_0(y_0+\Delta)$, which we expand in a Taylor series

$$J_0(y_0 + \Delta) = J_0(y_0) + \Delta J_0'(y_0) + \cdots \cong -\Delta J_1(y_0),$$

where second order terms have been discarded. Equation (31) yields at once the value for Δ,

$$\Delta = -\frac{u}{y_0}. \tag{32}$$

From (30) and (15) we get

$$y_0 + \Delta = a\sqrt{k_1^2 + h^2}$$

and so,

$$h^2 \cong -k_1^2 + \left(\frac{y_0}{a}\right)^2 + \frac{2y_0\Delta}{a^2} = -k_1^2 + \left(\frac{y_0}{a}\right)^2 + \frac{2\omega\epsilon_1 k_2}{a\sigma_2}. \tag{33}$$

Substituting the value of k_2 from (11) and using the abbreviation

$$w = \frac{\sqrt{2}\,\omega\epsilon_1}{a}\sqrt{\frac{\omega\mu_2}{\sigma_2}} \tag{34}$$

(33) may be written as

$$h = \left\{ i^2 \left[k^2 - \left(\frac{y_0}{a} \right)^2 + w \right] + iw \right\}^{1/2}. \tag{35}$$

This expression gives the value of h for a tube of finite high conductivity, but it may be simplified further by performing a binomial expansion and discarding terms of second and higher order. In this way we obtain

$$\left. \begin{aligned} h &= \alpha + i\beta, \\ \alpha &= \frac{\omega}{2\sqrt{k_1^2 - \left(\frac{y_0}{a}\right)^2 + w}} \\ \beta &= \sqrt{k_1^2 - \left(\frac{y_0}{a}\right)^2 + w} \end{aligned} \right\}. \tag{36}$$

Comparison of this value of h with that obtained for perfectly conducting tubes (18) shows a modification of the phase constant β and a nonvanishing attenuation constant α. They will now be discussed in detail for a tube with the dielectric properties of free space.

Attenuation constant α. We assume for the purpose of this discussion that the dielectric material inside the tube is sufficiently well characterized by $\epsilon_1 = \epsilon_0$, $\mu_1 = \mu_0$. It is convenient to put $k_1 = 2\pi/\lambda_e$ and take λ_e/a as the variable; after some rearrangement α may be put in the following form

$$\alpha = \frac{b}{a^{3/2}\sqrt{\frac{\lambda_e}{a} - \left(\frac{y_0}{2\pi}\right)^2 \left(\frac{\lambda_e}{a}\right)^3}} = a^{-3/2} bF$$

where,

$$\begin{aligned} b &= \sqrt{\frac{\pi}{c\mu_2\sigma_2} \cdot \frac{\mu_2}{\mu_0}} \\ F &= \left[\frac{\lambda_e}{a} - \left(\frac{y_0}{2\pi}\right)^2 \left(\frac{\lambda_e}{a}\right)^3 \right]^{-1/2}. \end{aligned} \tag{37}$$

In this expression, w, under the radical, has been neglected; therefore, it is not valid for λ_e very near the critical wave length λ_0. The nature of the metal of which the tube is made enters only in the factor b. The following table gives values of μ_2, σ_2, and b for several common metals:

TABLE I

Metal	σ_2 mhos/cm.	μ_2 henrys/cm.	b
Copper	5.8×10^5	1.257×10^{-8}	1.21×10^{-4}
Aluminum	3.3×10^5	1.257×10^{-8}	1.59×10^{-4}
Lead	4.81×10^4	1.257×10^{-8}	4.16×10^{-4}
Iron	1×10^5	1.257×10^{-6}	28.90×10^{-4}

Because α is inversely proportional to the three-halves power of the tube radius, the attenuation decreases rather rapidly with increasing tube size, λ_e/a being held constant. Fig. 9 shows graphically the factor $F=[(\lambda_e/a)-(y_0/2\pi)^2(\lambda_e/a)^3]^{-\frac{1}{2}}$ to which α is also proportional. There is an optimum ratio of wave length of the excitation to tube radius for which the attenuation will be a minimum. By equating to zero the derivative of F with respect to λ_e/a, we find that the optimum value is quite approximately

$$\frac{\lambda_e}{a}=1.5 \text{ for minimum attenuation.}$$

However, inspection of the curve of Fig. 9 indicates that the choice of λ_e/a for a given tube is not very critical; between the limits $1<\lambda_e/a<2$ the function F remains within ten per cent of unity. The correct operation of a hollow tube transmission system of the type described here obtains when the relation $1<\lambda_e/a<2$ is satisfied. In order to form a quantitative idea of the transmission through actual tubes, computa-

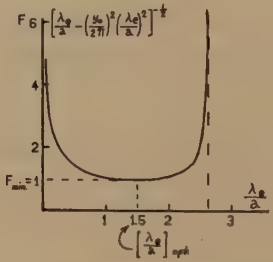

Fig. 9—Variation with λ_e/a of the attenuation in an air-filled copper pipe.

tions have been made for α (x measured in centimeters) and for the length of tube X ($X=1/\alpha\cdot10^{-5}$ in kilometers) traveled by the wave before it is attenuated to $1/e \cong 0.368$ of its original amplitude for the four metals listed in Table I. These data are tabulated in Table II. In Table III are shown the computed attenuation constants in decibels per mile. The values shown in both tables are for $F=1$, its minimum value; therefore they represent correct operating conditions $1<\lambda_e/a<2$ Under this condition, we find for *copper tubes*

attenuation $=170\ a^{-3/2}$ decibels per mile.

TABLE II

a cm.	α (x in cm.)				X in km.			
	Copper	Aluminum	Lead	Iron	Copper	Aluminum	Lead	Iron
2	$4.24 \cdot 10^{-5}$	$5.61 \cdot 10^{-5}$	$1.47 \cdot 10^{-4}$	$1.02 \cdot 10^{-3}$	0.23	0.18	0.07	0.01
5	$1.17 \cdot 10^{-5}$	$1.42 \cdot 10^{-5}$	$3.72 \cdot 10^{-5}$	$2.59 \cdot 10^{-4}$	0.93	0.70	0.27	0.04
10	$3.80 \cdot 10^{-6}$	$5.02 \cdot 10^{-6}$	$1.31 \cdot 10^{-5}$	$9.12 \cdot 10^{-5}$	2.6	2.0	0.76	0.11
20	$1.35 \cdot 10^{-6}$	$1.77 \cdot 10^{-6}$	$4.65 \cdot 10^{-6}$	$3.23 \cdot 10^{-5}$	7.4	5.7	2.2	0.31
30	$7.30 \cdot 10^{-7}$	$9.65 \cdot 10^{-7}$	$2.53 \cdot 10^{-6}$	$1.75 \cdot 10^{-5}$	14	10	4.0	0.57
40	$4.75 \cdot 10^{-7}$	$6.28 \cdot 10^{-7}$	$1.64 \cdot 10^{-6}$	$1.14 \cdot 10^{-5}$	22	16	6.1	0.87
50	$3.40 \cdot 10^{-7}$	$4.48 \cdot 10^{-7}$	$1.17 \cdot 10^{-6}$	$8.16 \cdot 10^{-6}$	30	22	8.5	1.2

For $1 < \lambda_e/a < 2$; error <10 per cent.

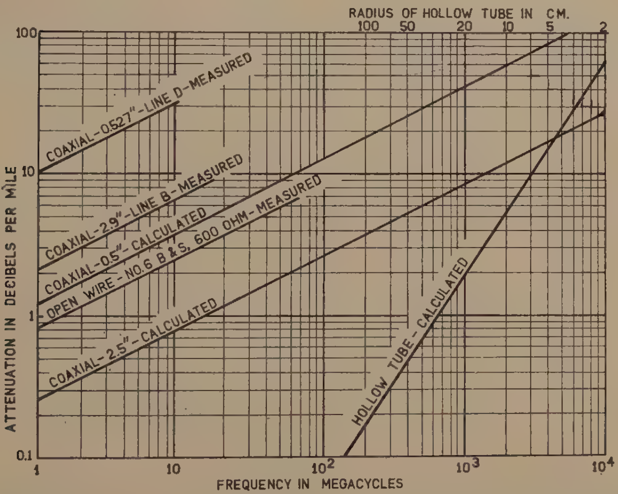

Fig. 10—Comparison of the attenuation in a hollow tube transmission system (air-filled copper pipe) with measured and with computed attenuations in several coaxial lines and an open-wire line. It is assumed that the hollow tube system is dimensioned for minimum attenuation.

TABLE III

a cm.	α in decibels per mile			
	Copper	Aluminum	Lead	Iron
2	60.	78.	210.	1430.
5	16.	20.	52.	360.
10	5.3	7.0	17.	130.
20	1.9	2.5	6.5	45.
30	1.0	1.4	3.5	25.
40	0.67	0.89	2.3	16.
50	0.48	0.63	1.6	10.

For $1 < \lambda_e/a < 2$; error <10 per cent.

A comparison of the attenuation in several open-wire and concentric-tube lines with that computed for the hollow tube system is presented graphically in Fig. 10. The curve for the hollow tube assumes that the radius is so changed with the frequency that the optimum

value $\lambda_e/a=1.5$ obtains at all frequencies. Data for the curves marked "Line B" and "Line D" and the expression

$$\text{db per mile} = 0.676\sqrt{f_{mc}}/b_{\text{inches}}$$

for computing the attenuation in coaxial lines were taken from a paper by E. J. Sterba and C. B. Feldman[16] (Figs. 2 and 11 of their paper). It appears that the hollow tube transmission system compares favorably, as to attenuation, with lines of conventional design. At the very high frequencies at which the hollow tube would be operated, it would probably have lower attenuation than any other type of line now in use, because the conductance loss could be reduced to the minimum obtainable by using an appropriate gas and pressure within the tube. In fact, the hollow tube transmission system is ideally suited to the "conduction" of waves of several centimeters length.

Phase constant β. Under the same assumptions as to the dielectric within the tube made in the preceding paragraph, we may write

$$\beta = \sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{y_0}{a}\right)^2 + w} \tag{38}$$

$$v_p = \frac{\omega}{\beta} = \frac{c}{\sqrt{1 - \left(\frac{y_0}{2\pi}\right)^2\left(\frac{\lambda_e}{a}\right)^2 + \frac{b}{\pi\sqrt{a}}\sqrt{\frac{\lambda_e}{a}}}}.$$

In the region of optimum attenuation $1<\lambda_e/a<2$, w is very small compared to the difference of the other two terms under the radical; consequently the phase constant β, and also v_p, v_g, and λ are not changed appreciably from their values (21) for perfectly conducting tubes. This includes cases of most practical interest. Near the critical frequency f_0, w is not negligible; the effect is to lower the velocity of propagation v_p compared to its value for perfect conductivity and to lower the value of the upper limit of the region of complete attenuation. This extension of the transmission region is greater for small tubes than for large ones, but the magnitude of the change is small for good conductors. Therefore, we may conclude that the phase constant is substantially the same for imperfectly conducting tubes as it was for perfectly conducting tubes; viz.,

$$\beta \cong \sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{y_0}{a}\right)^2}.$$

[16] E. J. Sterba and C. B. Feldman, *Bell. Sys. Tech. Jour.*, vol. 11, p. 411; July, (1932).

The Hollow Tube as a Filter

The property of transmitting only those waves whose frequencies lie above the critical value f_0 allows the hollow tube system to be used as a kind of high-pass filter. Viewed in this light, f_0 (19) becomes the cutoff frequency that separates the transmission band ($f>f_0$) from the attenuation band ($f<f_0$). It appears from the preceding anlaysis that the transmission loss is infinite throughout the attenuation band. In the ideal case of perfectly conducting tubes there would be no loss in the transmission band, but in actual tubes of finite conductivity there will be attenuation, which may be determined from α in (37). There will be a phase shift also, whose value may be secured from β in

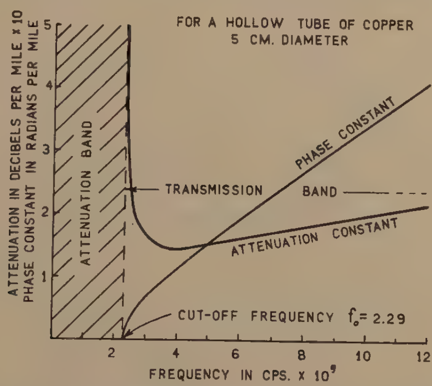

Fig. 11—Transmission characteristics for a five-centimeter radius hollow pipe of copper.

(38). In Fig. 11, curves of the attenuation and phase-shift characteristics are reproduced for a filter of this type comprising a copper tube of five centimeters radius. It is assumed that the tube is correctly terminated to avoid any reflections. Similar curves may be obtained for other metals and other radii. It may be observed that the attenuation drops to low values very rapidly as the cutoff frequency is exceeded, and that it increases quite slowly after reaching its minimum value at $f=f_{opt}$. Thus, a hollow tube of this size and material has an almost flat attenuation characteristic and a substantially linear phase characteristic over a band of about ten thousand million cycles per second width.

Because of their unusual properties, sections of hollow tube circuits might be used in connection with conventional lines and networks and with other hollow tube arrangements to obtain attenuation and phase-shift characteristics for band-pass, corrective, or other functions.

Terminal Devices

The preceding discussion was both quantitative and accurate, because the theory of the propagation of electromagnetic waves in hollow conducting tubes was firmly established from the Maxwell equations. We shall now consider a part of this transmission system; viz., the terminal device, that has not yielded to mathematical analysis and that, consequently, cannot be discussed in a quantitative and exact manner. Experiment must be invoked to verify the anticipated behavior of a given terminal device. With this caution, we turn our attention to

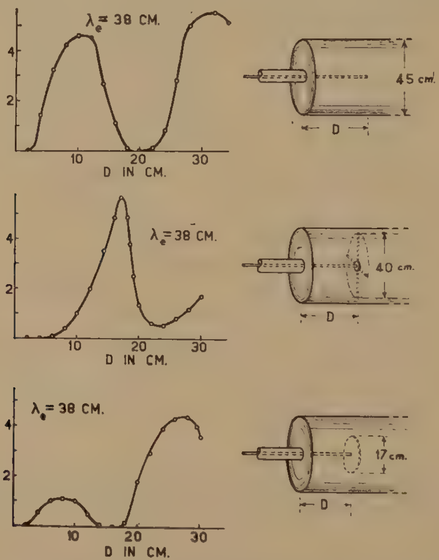

Fig. 12—Three types of terminals for a hollow tube transmission system for waves of the longitudinal type. The associated curves show field intensity at far end of pipe vs. the length D of the coaxial rod. These curves illustrate the adjustment of the terminal for maximum energy transfer, and they show the sharply resonant properties of these terminals.

means for exciting waves in the hollow tube at the sending end, for intercepting them at the receiving end, and for radiating them directly into space at the far end of the tube.

A terminal device at the sending end must take high-frequency energy from a pair of conductors and cause this energy to be propagated down the hollow tube. From the circuit point of view, this terminal device must connect the biconductor exciting system with the uniconductor transmission system. Except for questions of insulation and of impedance of the biconductor circuit, terminal devices for the sending and for the receiving ends may be identical, because any device that

will radiate waves of the hollow tube type will be equally effective in picking them up.

The hollow pipe transmission system differs radically from conventional systems in regard to the configuration of the field about the conductor. In conventional transmission lines only one type of wave can be propagated. In the hollow pipe system, on the other hand, several distinct kinds of waves are possible, as indicated in connection with (4). Consequently, the terminal device may be designed to excite waves of the desired type to the exclusion of those of other types. Such design is advisable if good efficiency is to be secured. Generally, both sending and receiving terminals should transmit and receive waves of a single type only. In addition to efficiency, this allows a unique kind of multiplexing to be described later.

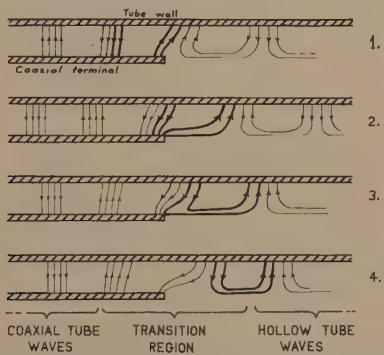

Fig. 13—Illustrating probable shapes of the lines of electric intensity at a coaxial type terminal for successive instants of time.

Terminals for sending and receiving the previously discussed waves of several different configurations have been used. The simplest and one of the most effective comprises a length of coaxial rod fed in any of several ways. It is shown with coaxial feed at the top of Fig. 12. Because of the symmetry of its electromagnetic field, the coaxial rod terminal is well suited to excite or to intercept hollow tube waves. A qualitative idea of the way in which it operates may be obtained from the series of sketches in Fig. 13, in which the lower half of the tube section is omitted for economy of space. The waves travel faster in the hollow tube than they do in the coaxial terminal, and the wave lengths in the two portions of the tube are approximately related by the inequality

$$\lambda_{\text{hollow tube}} = \frac{\lambda_e}{\sqrt{1 - \left(\frac{y_0}{2\pi}\right)^2 \left(\frac{\lambda_e}{a}\right)^2}} > \lambda_e \approx \lambda_{\text{coaxial terminal}}.$$

Hence, we may consider that the field is distorted to allow the formation of closed loops in the electric field that break away from the coaxial rod terminal and propagate themselves down the hollow tube. The action of this coaxial rod is interesting when compared to the radiation from a rod in free space, inasmuch as in the latter case absolutely no radiation takes place in the direction of the axis of the rod.

We may for convenience think of this arrangement as a connection of a concentric tube to a hollow tube and make use of the conception of impedance matching. The magnitudes of the several dimensions of the terminal would then be made such that the impedance looking into the terminal under operating conditions is matched to that into the external circuit. This consideration also applies to the other terminal devices. The experimentally determined curve shown with the sketch in Fig. 12 illustrates the adjustment of the terminal for maximum energy transfer to the tube. It was obtained by measuring the maximum field intensity at the receiving end of the tube as the coaxial rod was slid in or out. Clearly, a much better energy transfer may be secured by correctly adjusting the terminal device.

Several modifications of this terminal device are shown in Fig. 12. By adjusting the relative dimensions of the rod, the maximum energy transfer from the source into the tube can be accomplished in each case. In the second terminal shown, radial wires are employed to concentrate the radial current and allow reflections from the back face to be given the proper phase to reinforce the forward radiation, in about the same way that a reflector is used in a directive radiating system. This terminal may be made to exhibit exceedingly sharp resonance by increasing the number of radial wires. The third sketch shows a coaxial rod with a metal disk on its end, together with its matching curve. All of these terminals may act as more or less sharply resonant systems. They may be "tuned" and utilized in hollow pipe systems, in much the same manner as resonant combinations of elements are used in conventional circuits. They may also be thought of as electromagnetic analogues of the Helmholtz resonator.

The vacuum tube oscillator or the detector may be placed entirely within the pipe in any of these terminal devices by inserting it at about the middle of the coaxial rod, a section of which may be replaced by an insulator, the oscillator terminals being connected to the two adjacent ends of this rod. The coaxial rod is preferably made hollow and the filament, grid, plate, etc., leads brought to the vacuum tube through its interior.

Experimental Tests

Experimental work has been carried out with a galvanized iron cylinder (an old air duct) approximately one and five-tenths feet in

diameter by sixteen feet long. A Barkhausen oscillator operating at free-space wave lengths from about thirty-eight centimeters upwards has been the main source of power. It was modulated at a low audio frequency to allow the simplest kinds of receivers to be used for measuring the field. Either a crystal detector or an acorn tube with short rods as an antenna proved to be a thoroughly satisfactory "probe" for the field measurements; an amplifier, a copper-oxide meter, and headphones completed the receiver. The general scheme of an experimental setup is indicated by Fig. 14, where a parallel wire type feed for the sending terminal is shown. The probe stick was held in the hands or mounted in guides for more accurate alignment.

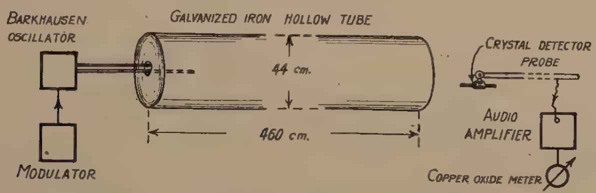

Fig. 14—Diagram of typical experimental pipe and apparatus.

For many practical applications, a pipe diameter of only several inches is contemplated, necessitating an excitation with a free-space wave length of the same order of magnitude. The difficulties of experimenting with such "centimeter waves" has been obviated here by using a *large scale model* of the proposed device; most of the measurements so obtained may be applied unambiguously to the smaller pipes. It is interesting to observe that we have already reached the point in ultra-high-frequency work where the principle of models may be reversed in the above manner with advantage. However, we may not be able to go much further with this idea, and even here we must exercise caution, because for the lower "centimeter waves" the molecular and atomic natures of gases, liquids, and solids, begin to affect materially the behavior of our electromagnetic systems.

The critical free-space wave length for the above-mentioned pipe was about sixty centimeters, which was easily verified; no transmission took place when the excitation exceeded this value, even when the power was increased to about twenty-five watts at one and five-tenths to two meters. Thus, the critical frequency and the filter action of the hollow tube were verified.

By moving the probe along the inside of the tube, pronounced standing waves were obtained, as illustrated by Fig. 15. These measurements were made by sliding the probe, coaxially oriented in guides, along the axis of the pipe at the receiving end. The effect of probe length is shown

by the three curves. The effect of stray pickup (zero-centimeter probe) is negligible. The shape and magnitude of the curves depend on the probe length; a length of fifteen centimeters was used in most of the measurements. It is not to be expected that the curves go to zero at the

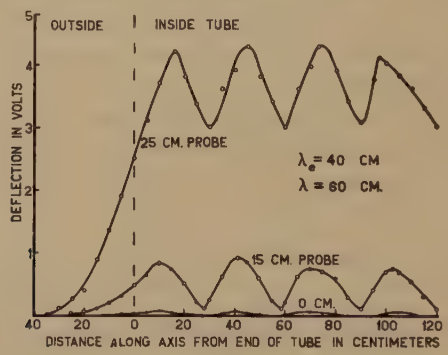

Fig. 15—Standing waves measured along the axis of a hollow tube with coaxially oriented probe. The effect of probe length on the measurements and the stray effects (zero-centimeter probe) are shown by the three curves.

minimums, because an open-ended hollow tube, unlike the ordinary unterminated transmission line, radiates rather effectively, thereby placing an appreciable load on the receiving end. Similar curves have been taken for both open- and closed-end pipes for a variety of conditions. Among other things, they allow a rather precise determination

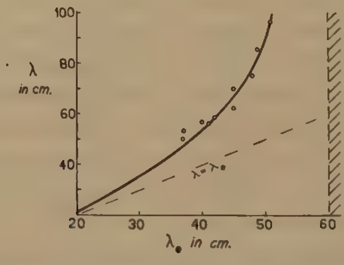

Fig. 16—Wave length λ inside pipe vs. free-space wave length λ_e of the excitation; circles indicate measured values, the solid curve is computed from the theory.

of the wave length within the tube and an evaluation of the degree of matching obtained with different terminations. With certain terminations the standing waves have been substantially absent, indicating that the tube was correctly terminated in its characteristic impedance.

Fig. 16 shows measured values of the wave length in the tube λ

(small circles) compared to the calculated values (solid curve), both plotted vs. the corresponding wave length λ_e of the excitation, as measured on Lecher wires at the transmitter. The agreement is quite satisfactory for these relatively crude measurements. The hollow tube wave length is greater than that of the excitation by the predicted amount, hence this also verifies the theoretical value for the velocity of phase propagation, which is greater than light.

Speech and other signals, have been satisfactorily transmitted through the pipe and received at the far end with amounts of power so small that free-space radio reception, using the same transmitting and receiving apparatus, was only possible over about one tenth of the pipe length. As yet no figures are available on the attenuation in the tube.

Transversely Polarized Waves

We have already pointed out the possibility of entirely different kinds of waves in the hollow tube having field components H_x, H_r, and E_ϕ. In these waves, the electric field is confined to planes transverse to the axis and they resemble, to a certain degree, plane waves propagated along the tube, because there is no longitudinal component of E. The boundary condition for this set of waves, referred to by Rayleigh as "vibrations of the second class," requires that $H_x(a)=0$, that is, $J_n{}'\,(a\sqrt{k_1^2+h^2})=0$. He has also discussed the fact that the type of wave having the lowest critical frequency of all possible hollow tube waves is that corresponding to the first root for $n=1$ and it is to this type of wave that we now direct our attention. The critical frequency is given by

$$f_0 = \frac{1.841}{2\pi a\sqrt{\mu_1\epsilon_1}} \tag{39}$$

and the phase constant for perfectly conducting tubes by

$$\beta = \sqrt{\omega^2\mu_1\epsilon_1 - \left(\frac{1.841}{a}\right)^2}. \tag{40}$$

For air-filled tubes, these expressions give rise to the special forms

$$\left.\begin{aligned} \lambda_0 &= \frac{2\pi a}{1.841} = 3.41a \\ \beta &= \sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{1.841}{a}\right)^2} \quad \text{(real)} \\ \lambda &= \frac{a}{\sqrt{\left(\frac{a}{\lambda_e}\right)^2 - 0.086}} \quad \text{(real)} \end{aligned}\right\}. \tag{41}$$

These several quantities have been adequately verified by experiment. The attentuation in finitely conducting pipes may be calculated by the method already used for the longitudinal type of wave.

Terminal devices for this transverse wave are fundamentally different in construction from those of Fig. 12. Again, the configuration of the wave that is to be excited determines the design of the terminal and in this case the simplest result is obtained by employing a rod at right angles to the axis across a diameter of the pipe. In this position it coincides with a line of electric intensity for the transverse wave. This rod may be fed in one of several ways, the sending end of the tube is preferably closed, and the whole device may be dimensioned to effect

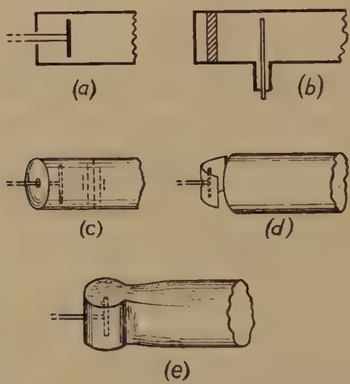

Fig. 17—Several types of terminals for a hollow tube transmission system for waves of the transverse type.

a maximum energy transfer into the tube or used as a resonant device, as previously described for the longitudinal waves and as illustrated in Fig. 12. Several terminals of this kind are reproduced in Fig. 17. In (a) is shown a parallel wire feed and in (b) a coaxial feed. Additional parasitic or driven rods or one or more wire grids may be inserted down the tube, as indicated in (c), to reinforce the wave much as wave directors are used. A parabolic reflector as in (d) has been used with considerable success; the parabola may be placed entirely within the pipe. The length of the rod may be adjusted to optimum operating conditions without affecting the wave type, even to the extent where the pipe walls are bridged by it. A length equal to $\lambda_c/2$ has been used in much of the experimental work. The vacuum tube may be placed entirely within the pipe and made an integral part of the rod construction. In (e) is shown a cylindrical resonator attached to the hollow tube system; the radiation characteristic of this resonator has been investigated separately by Sloat[17] under the author's supervision.

Measurements with the probe of the field distribution over the pipe cross section and of the polarization of the waves within the tube have demonstrated that the transverse type of wave can be excited alone with terminals of the above description. Similarly, it was shown that the longitudinal type of wave alone can be excited with the coaxial type of terminal.

Multiplex Transmission

An important property of transmission by means of the transverse wave is the fact that there is one orientation of the receiving terminal rod, under ideal conditions at right angles to its sending terminal rod, for which zero voltage is induced in it. Also, there is another orientation substantially parallel to the sending terminal rod, for which the re-

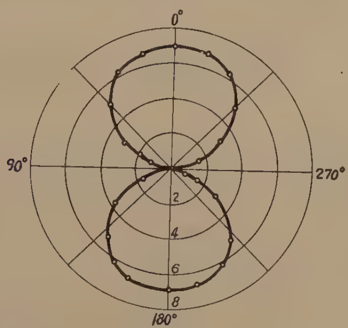

Fig. 18—Received signal strength vs. probe orientation for transverse waves polarized in 0-to 180-degree plane.

ceived signal is a maximum. A curve showing this effect is reproduced in Fig. 18. This curve was experimentally obtained by plotting the deflection, which is closely proportional to the electric field in intensity, in polar co-ordinates as the probe was rotated about the axis in a plane perpendicular thereto. The orientation of the sending terminal rod was 0 to 180 degrees (vertical); the maximum signal was received with the same (vertical) orientation, and no signal was received at the 90- and 270-degree (horizontal) orientation.

Equally important is the fact that when the symmetry of construction is sufficiently good, a receiving terminal for the transverse waves will not respond to waves of the longitudinal type, and vice versa. Inasmuch as several distinct types of waves may be excited and transmitted simultaneously, which are mutually independent and which may be separately and independently intercepted at the receiving end

[17] J. R. Sloat, Master's thesis in electrical engineering, Massachusetts Institute of Technology, June, (1934).

of the system, the hollow tube transmission system may be operated in a unique kind of multiplex. A terminal device for multiplex operation is shown in Fig. 19, where three separate rods are fed coaxially and provide three independent channels for communication within the same pipe. Rod 1 provides one channel using the longitudinal waves, rod 2 a second channel using vertically polarized transverse waves, and rod 3 a third channel using horizontally polarized transverse waves. This multiplex operation, which is not thought possible on biconductor transmission lines, and which may be capable of further expansion, considerably increases the possible usefulness of the hollow tube system.*

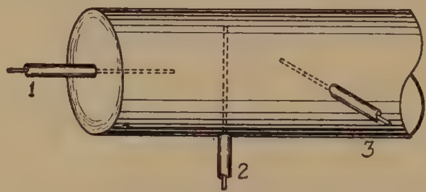

Fig. 19—Terminal device for multiplex operation of a hollow tube transmission system. One longitudinal (1) and two transverse (2 and 3, at right angles to each other and to 1) types of waves provide three communication channels in the same pipe.

Radiation from the Tube End

As indicated in the Introduction, the far end of the tube may be made into a radiator, instead of a translation device from the uniconductor pipe to a biconductor system. The open end of the pipe forms the simplest arrangement. Electromagnetic energy flows from the pipe end and is propagated into space as radio waves. Not all of the energy is radiated, because of reflections from the end, and standing waves are set up in the pipe. Since the pipe cross section is comparable in dimensions to the wave length, a radiation pattern having directive characteristics is produced. The shape of this pattern depends on the ratio λ_e/a and on the type of the wave.

Fig. 20 shows the measured pattern obtained with a wave of the longitudinal type. The pipe was parallel to the ground and two feet above it. The probe was oriented for maximum deflection in a horizontal plane through the axis of the pipe and the distance at which the deflection was constant was plotted. It is realized that this method is not strictly correct, but the pattern so obtained is thought to be indicative of the actual free-space pattern. It may be demonstrated by a

* Note added in proof. Two-channel multiplex operation was demonstrated before the Boston Section of the I.R.E., May 22, 1936.

qualitative argument that the radiation pattern should have this general shape. The pattern in free space will be symmetrical about the axis of the tube and the wave propagation may be reasonably compared

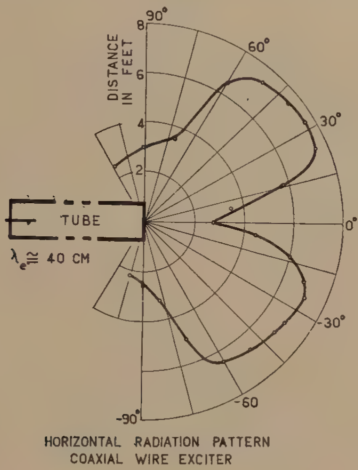

Fig. 20—Radiation pattern from open end of pipe with longitudinal type waves.

with the blowing of smoke rings through a tube. Somewhat similar results were reported by Bermann and Kruegel.[12]

If waves of the transverse type are employed, the radiation pattern is entirely different, having a maximum to the front. Fig. 21 shows a

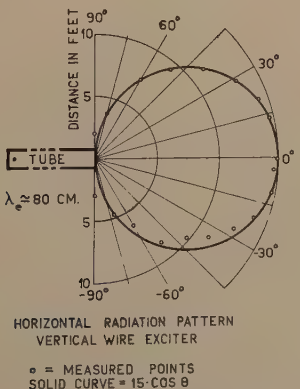

Fig. 21—Radiation pattern from open end of pipe with transverse type waves.

pattern measured with the same arrangement as before, except that the probe was kept vertical; i.e., parallel to the sending terminal rod. In this figure, the solid curve is a plot of cos θ, hence it appears that

for the conditions of the measurement (λ_e/a was almost at the critical value) the pattern is given by this simple function. Transverse waves appear to be well suited for producing a beam type of pattern.

Electromagnetic Horns

In order to match properly the hollow pipe to the external space and to produce more directive patterns, the pipe can be flared into a horn-shaped radiator. The analogy of this kind of radiator, which is believed to be new, to an acoustic horn is indeed close. However, the throat of an acoustic horn is usually much smaller than the wave length while the throat of the hollow tube horn is comparable to it.

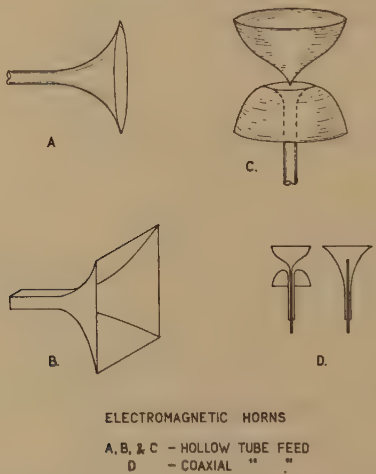

Fig. 22—Several types of electromagnetic horns.

The radiation patterns of these horns are affected by the same factors that influence the open-end pipe, viz., λ_e/a and the wave type, and in addition by the shape of the horn proper. In Fig. 22 several electromagnetic horns are illustrated. *A* and *B* show simple flared types of circular and of rectangular cross section, respectively, and C shows an interesting type for longitudinal waves that produces a uniform radiation concentrated in a plane perpendicular to the axis of the tube. Thus, A and C are adaptable to broadcast radiation with longitudinal waves and A and B to beam radiation with transverse waves.

The application of horn radiators is not confined to the hollow tube system, for they may be fed by a coaxial or other lines. This application is illustrated by Fig. 22, D. Thus, electromagnetic horns may be used as radiators in the wave band below ten meters where their dimensions may be kept reasonably small.

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A CRITICAL STUDY OF TWO BROADCAST ANTENNAS*

By
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Summary—*A theoretical study is made of a double doublet, determining the vertical radiation characteristics for various current ratios. Also, the horizontal field intensity is plotted as a function of the current ratio and phase angle. Three experimental designs are discussed. Fading measurements show the improvement of the upper doublet in comparison with a quarter-wave antenna. A theoretical study is made of a loaded quarter-wave antenna showing vertical radiation characteristics and horizontal field intensity as a function of the current distribution. Recording measurements verify that fading at a distance is materially reduced when the antenna is properly loaded.*

Introduction

THE problem confronting the radiation engineer is to furnish to a maximum number of potential listeners a satisfactory field intensity for the operation of their receivers. For moderate powered transmitters the primary service area is by far the most important from a commercial standpoint. Since interfering stations are usually located in the secondary service area it would be a distinct advantage to minimize the high angle radiation and put this energy into the ground wave. This would reduce station interference and increase the primary service area of the station by pushing back the fading wall and increasing the primary service area field intensity to override local interference.

Since extreme heights are not very practicable and in many instances cannot be obtained because of the hazard to air navigation, a critical study was started, in the fall of 1934, on several antennas which will operate within a quarter wave length. The purpose of this paper is to discuss two such antennas and show some of the experimental results obtained from the investigation.

Double Doublet Theory

In making a theoretical study of the double doublet, the following assumptions were made: oscillating doublets of infinitesimal length along the radiator, sinusoidal current distribution, a perfect conducting earth, an absence of power sinks above the earth, that the folded portions have no effect upon the vertical radiation pattern, and

* Decimal classification: R320. Original manuscript received by the Institute, June 15, 1936. From a thesis presented to the Ohio State University in partial fulfillment of the requirements for the Professional Degree of Electrical Engineer.

a current loop exists at the center of each doublet. This last assumption was not realized in the first experimental design but was obtained in the second experimental design.

The Poynting vector method was employed to determine the field intensity as a function of the phase angle ϕ and the current ratio γ between the current loops on the upper and lower doublets, as shown[1] in Fig. 1. The absolute magnitude of the field intensity can then be written,

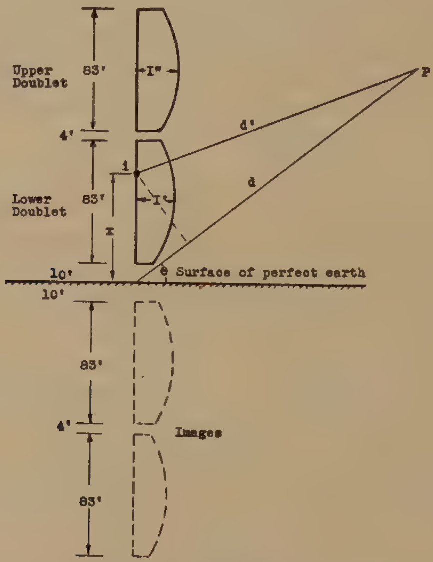

Fig. 1—Specific dimensions of double doublet used in this theoretical analysis. $f=1390$ kilocycles.

$$/E_\theta/ = \frac{60}{d}\sqrt{\frac{P}{R}}\sqrt{[f'(\theta) + \gamma f''(\theta)\cos\phi]^2 + [\gamma f''(\theta)\sin\phi]^2} \qquad (1)$$

where,

$/E_\theta/$ = the field intensity in volts per meter
d = the distance from antenna in meters
θ = the elevation angle
P = the watts antenna power
$f'(\theta)$ = a function of θ for the lower doublet
$f''(\theta)$ = a function of θ for the upper doublet
γ = the current ratio of the loop current in the upper doublet to the loop current in the lower doublet

[1] C. E. Smith, "A critical study of several antennas designed to increase the primary coverage of a radio broadcasting transmitter." This is an unpublished thesis filed in the Ohio State University Library.

ϕ = the phase angle between the current in the upper and lower doublets

and,

$$R = 19.7 + 34.7\gamma \cos\theta + 15.6\gamma^2 \qquad (2)$$

is the radiation resistance of the double doublet in ohms referred to the loop of the lower doublet.

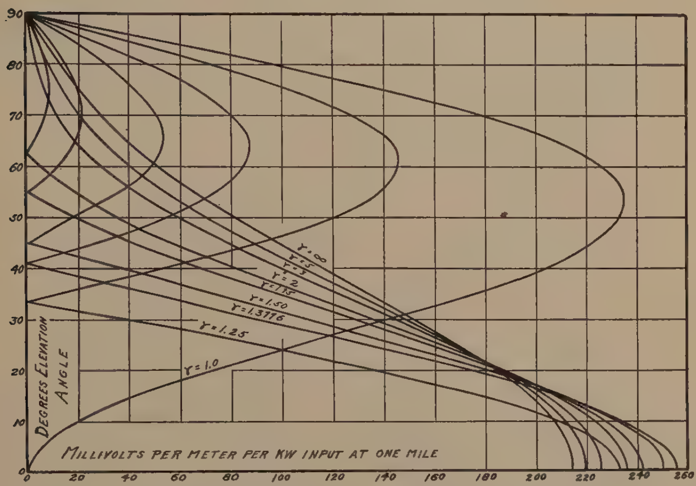

Fig. 2—Vertical radiation characteristics of double doublet for various values of the current ratio γ when the phase angle $\phi = 180°$.

If $/E_\theta/$ is maximized with respect to the phase angle ϕ and the current ratio γ, it is found that the ground wave

$/E_{\theta=0}/ = 255.5$ millivolts per meter per kilowatt input at one mile, when,

$\phi = 180°$ and

$\gamma = 1.3776$.

A family of vertical radiation characteristics have been plotted in Fig. 2 from (1) when $\phi = 180°$. For $\gamma = \infty$ the upper doublet is operating alone. As the current ratio is decreased the high angle radiation decreases and a small high angle lobe appears for $\gamma = 2$. This lobe increases considerably for maximum ground wave when $\gamma = 1.3677$. As the current ratio is decreased to 1.0 the lobe increases to a maximum, and the ground wave disappears.

The ground wave has been plotted in Fig. 3 as a function of the current ratio when $\phi = 180°$. Also, the lobe intensity has been plotted to show that with a little sacrifice in ground wave by operating to the right of the maximum, the lobe intensity is materially decreased.

In Fig. 4 the solid contour surface shows how the ground-wave intensity varies as a function of both the phase angle ϕ and the cur-

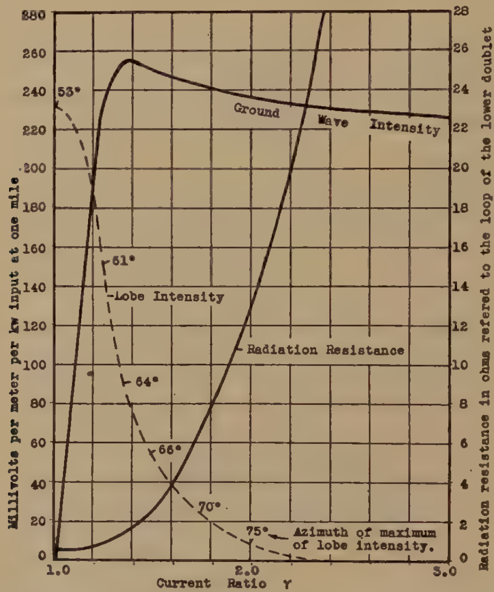

Fig. 3—Field strength of ground-wave and lobe and the radiation resistance as a function of the current ratio γ when the phase angle $\phi = 180°$.

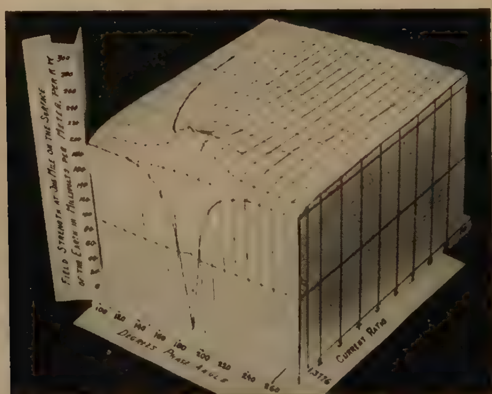

Fig. 4—Solid contour surface of ground-wave intensity as a function of the current ratio γ and phase angle ϕ.

rent ratio γ. The surface shows a rather sharp peak for the maximum ground wave. The curve of maximum ground-wave intensity for various values of phase angles ϕ has been drawn on top of the contour

surface, showing how the current ratio increases when the phase angle ϕ is made greater or less than 180°.

Two other current distributions were assumed inasmuch as practically all the work of computation was at hand. These current distributions, although impracticable to obtain, do give slightly higher theoretical maximum ground-wave intensities with a very slight change in the vertical radiation characteristic. The results are as follows: Current node at center of lower doublet with a current loop at the center of the upper doublet gave $/E_{\theta=0}/=257$ millivolts per meter per kilowatt input at one mile, and current node at the center of the upper doublet with a current loop at the center of the lower doublet gave $/E_{\theta=0}/=263$ millivolts per meter per kilowatt input at one mile.

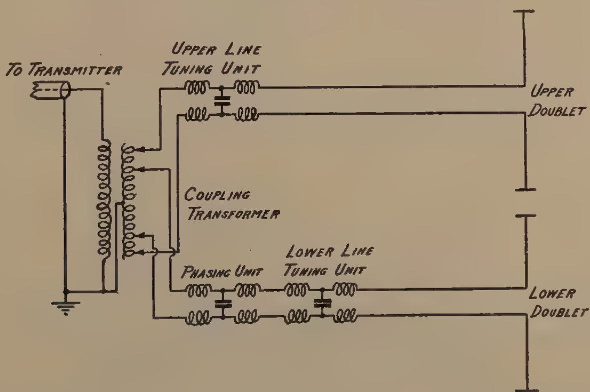

Fig. 5—First experimental design of double doublet.

Double Doublet Experiment

The first experimental design shown in the schematic diagram of Fig. 5 was suggested by J. C. McNary. The ends of the doublets were loaded with twenty-five feet of conductor arranged in a star shape. With this arrangement current loops are not realized at the center of the doublets and standing waves exist on the transmission lines. In order to make rapid changes in the phasing and line-tuning units a derivation was made[2] to determine the coil and condenser adjustments as a function of the phase shift in the unit. The equations are,

$$\frac{X_L}{4}=\frac{Z_0(1-\cos\beta)}{2\sin\beta} \tag{3}$$

$$X_c=\frac{Z_0}{\sin\beta} \tag{4}$$

[2] C. E. Smith, "Phase shifting networks," *Comm. and Broadcast Eng.*, vol. 3, p. 21; May, (1936).

where,

$X_L/4$ = ohms inductive reactance of one coil in the balanced H section

X_c = ohms capacitive reactance of the condenser

Z_0 = ohms characteristic impedance (resistance)

β = degrees phase shift of the H section

The experimental results with this system of feeding the doublets indicated that the doublets themselves are unbalanced to ground, especially the lower half of the lower doublet, the copper wire resistance loss is quite high due to the low radiation resistance of the doublets. Transverse currents set up in the feeder lines causes radiation which

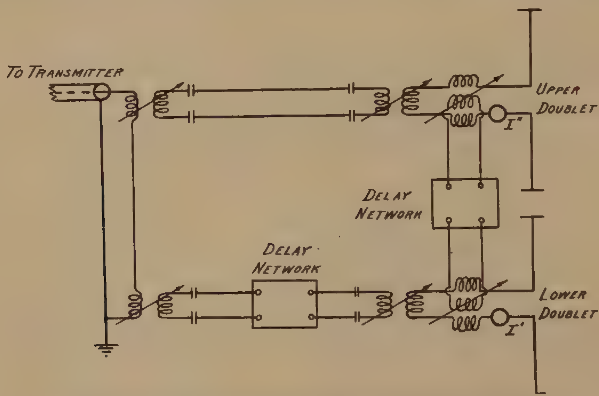

Fig. 6—Second experimental design of double doublet.

has a tendency to polarize the radiated waves into nonvertical planes. Doublet unbalance and mutual impedance between the doublets shift the current and voltage nodes along the feeder lines and hence the phase shift and current ratio between the doublets are uncertain.

The feeder system should be changed in such a way that the losses in the feeder lines will be a minimum and changing one doublet current will not affect the other doublet current.

The second experimental design shown in the schematic diagram of Fig. 6 was suggested by W. L. Everitt. A pictorial view of the antenna in Fig. 7 shows one of the towers which is broken into insulated sections thus minimizing the effect of the tower.

The link circuit is adjusted in such a way that coupling due to the antennas is balanced out. With this adjustment it is possible to drive one doublet without producing current in the other doublet. The experimental work verified that this scheme was practicable but still other difficulties had to be overcome.

Experimental evidence showed that a capacity ring connected to its center by many radials gave good loading for the ends of the dou-

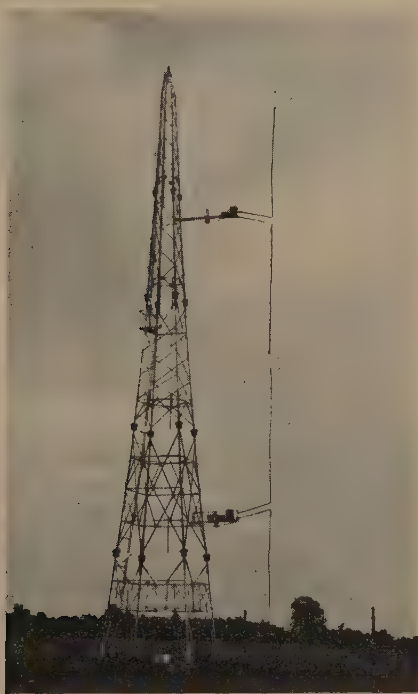

Fig. 7—Photograph of east tower showing the double doublet with the necessary guy ropes, catwalks, and tuning units. Note how the tower is broken up into four insulated sections.

blets, without corona as was experienced in the original design. In order to obtain current loops at the center of the doublets, to correspond with

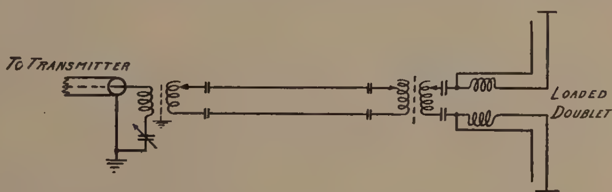

Fig. 8—Schematic diagram for feeding upper doublet.

the theoretical work, transmission lines terminated in inductive reactance elements as shown in Fig. 8 were used.[3]

[3] Nickle, Dome, and Brown, Proc. I.R.E., vol. 22, pp. 1362–1373; December, (1934).

Critical measurements were made on the upper doublet operating alone. It was found that shorting the insulated tower sections had a very considerable effect upon the radiation resistance and radiation properties of the doublet, Efficiency, based upon field strength measurements compared to the regular quarter-wave antenna, were as follows:

Line transformer and transmission line to the upper doublet	90.5 per cent
Doublet efficiency including the loading lines and coils	79.0 per cent
Over-all efficiency	71.5 per cent

The radiation resistance of the upper doublet for this condition was 11 ohms as compared to 15.6 ohms for the theoretical radiation resistance of this doublet over a perfect earth and not in the vicinity of a tower.

Operating the upper doublet on the basis of radiating one kilowatt, recording measurements[4] were made at eighteen points in various directions and at various distances from the transmitter. Alternate periods of fifteen minutes were devoted to the doublet and quarter-wave antenna. Some typical fading measurements are shown in Fig. 9. These measurements show that fading is more severe on the quarter-wave antenna at all distances, the average field strength is greater in the sky-wave area on the quarter-wave antenna than on the upper doublet, the average field strength is greater on the upper doublet in the primary service area than on the quarter-wave antenna, and the fading increases on both antennas as the distance from the transmitter is increased.

The results of these experiments indicate that the double doublet cannot be put into permanent operation with the existing losses and extremely low radiation resistance when operating to give a good vertical radiation pattern. However, if a power rating based upon the theoretical analysis could be obtained, as was done to obtain the above fading measurements, this type of antenna might be worth while even though the antenna efficiency is quite low.

Loaded Quarter-Wave Antenna Theory

The same assumptions were made in this analysis as were made on the double doublet. The absolute magnitude of the field intensity can be written,

[4] E. L. Gove and C. E. Smith, "Field strength measuring equipment," *Comm. and Broadcast Eng.*, vol. 2, pp. 12–15; October, (1935).

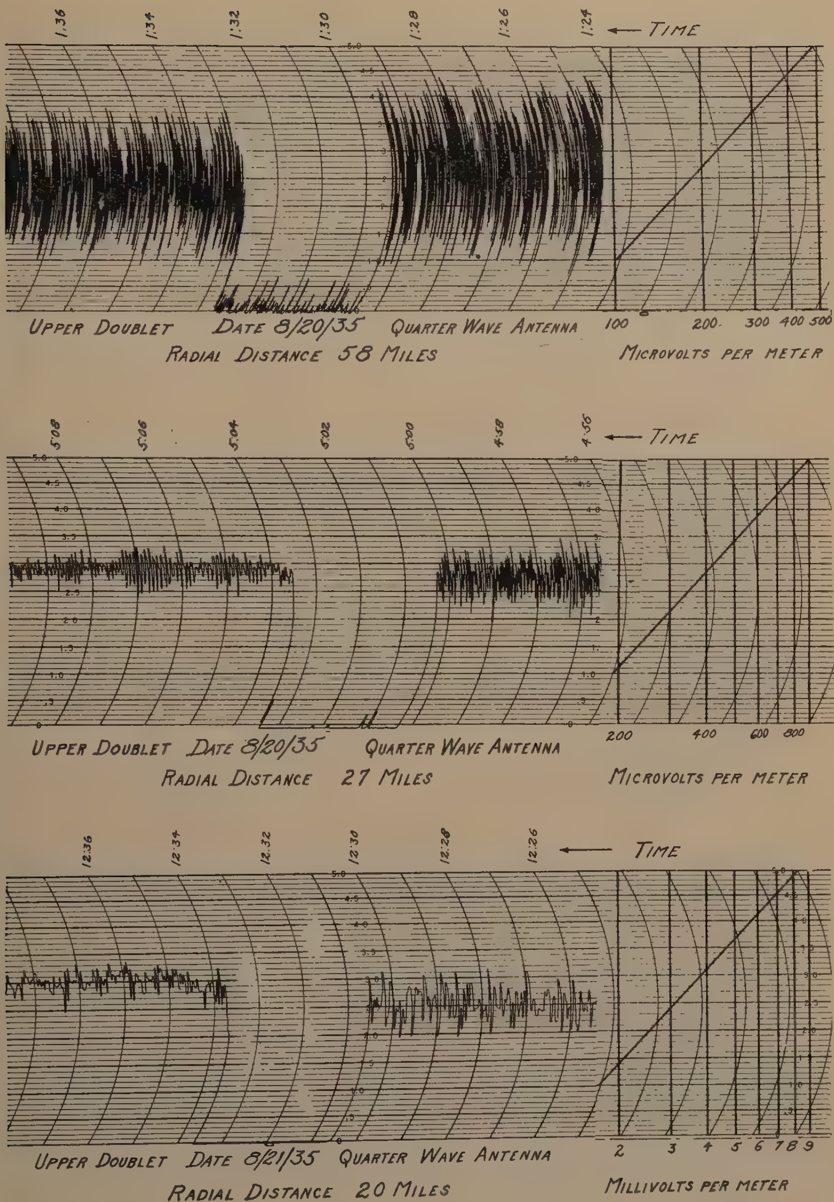

Fig. 9—Fading measurements made on upper doublet and quarter-wave antenna when radiating one kilowatt.

$$/E_\theta/ = \frac{60}{d} \sqrt{\frac{P}{R}}\ \frac{\cos\alpha \sin\theta \sin\left[(\pi/2)\sin\theta\right] - \cos\alpha - \sin\alpha\left[\cos(\pi/2)\sin\theta\right]}{\cos\theta} \quad (5)$$

where,

α = the distance from the surface of the earth down to the extended current node

$R = 32.42\cos^2\alpha + 34.29\sin 2\alpha + 36.56\sin^2\alpha$ (6)

is the radiation resistance of the loaded quarter-wave antenna in ohms referred to the current loop.

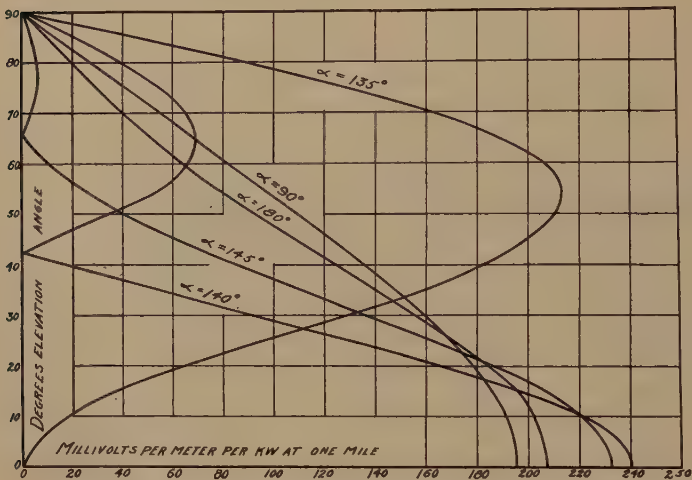

Fig. 10—Calculated vertical radiation characteristics of loaded quarter-wave antenna for various values of loading.

The other symbols are defined in (1). This equation is a special case of a more general form given by Brown.[5] The evaluation of the radiation resistance was made by series expansion of the various functions to eight significant figures. The curves obtained by this evaluation are not as optimistic as those published by Brown who obtained his results for maximum ground-wave by comparing the vertical radiation characteristic with that of a vertical 230° antenna and found the curves to be practically identical.

A family of vertical radiation characteristics have been plotted from (5) in Fig. 10. The curve marked $\alpha = 90°$ is for a standard quarter-wave antenna. If the current distribution is inverted the curve $\alpha = 180°$ is obtained, showing a slight increase in the ground wave and some

[5] G. H. Brown, Proc. I.R.E., vol. 24, p. 54; January, (1936).

decrease in the amount of high angle radiation. For $\alpha = 135°$ the current node is half way up the antenna giving a complete cancellation of the ground wave and a very large lobe. As α is increased to 140° the ground wave becomes a maximum with an appreciable high angle lobe. At $\alpha = 145°$ the high angle radiation is quite low with only a slight decrease in the ground wave. This should be a good condition of operation to minimize fading.

The horizontal field intensity and radiation resistance are plotted in Fig. 11 as a function of α. For maximum ground wave the radiation resistance is extremely low. As α is increased slightly the radiation resistance increases to more reasonable values for the practical operation of the antenna.

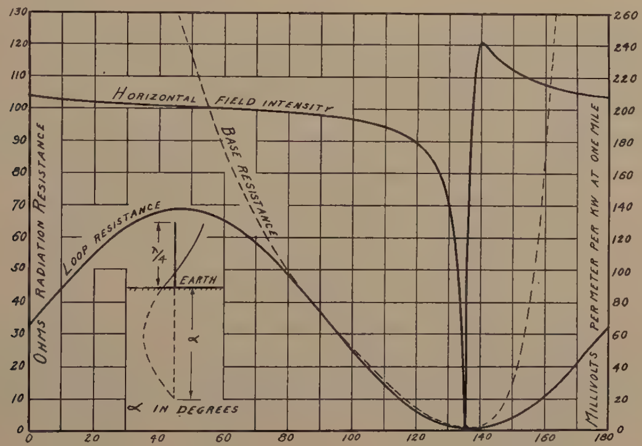

Fig. 11—Calculated horizontal field intensity of the ground-wave and the radiation resistance of a loaded quarter-wave antenna as a function of the current distribution expressed in terms of α.

Loaded Quarter-Wave Antenna Experiment

The recording shown in Fig. 12 verifies that the fading is materially reduced when the antenna is properly loaded. The loading capacity of 0.01 micromicrofarad should give a vertical radiation characteristic corresponding closely to the theoretical condition of $\alpha = 145°$.

Since the radiation resistance is quite low for both conditions of loading it is to be expected that the efficiency on the experimental model will also be low. Measurements show that the radiation efficiency as compared with the standard quarter-wave antenna is only 43.5 per cent for the first condition of loading and 36.2 per cent for the second condition of loading. This low efficiency is due primarily to copper loss

in the method used to load the antenna. By proper design this efficiency can be materially improved.

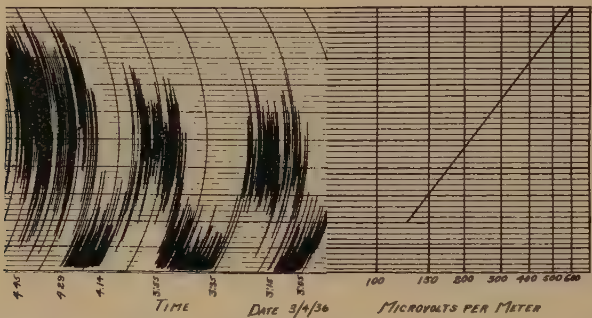

Fig. 12—Fading measurements made on a loaded quarter-wave antenna.
Radial distance from transmitter—53 miles
Receiving antenna—20-foot vertical—tuned.
Horizontal field strength near the transmitter is the same for each condition of operation
The power input to the standard quarter-wave antenna is one kilowatt by direct measurement.
3:15–3:35 loading capacity = 0.0065 micromicrofarad
3:55–4:14 loading capacity = 0.01 micromicrofarad
4:29–4:45 standard quarter-wave antenna

Conclusions

This investigation indicates that the condition of maximum ground wave is in general accompanied by a large high angle lobe for short vertical radiators. The best condition of operation is then, not to obtain a maximum ground wave but to minimize the high angle radiation. Short vertical radiators usually have a very low radiation resistance when a good vertical radiation characteristic is obtained.

The ground wave can be increased as the height of the antenna is increased. Care must be exercised in selecting a vertical radiation characteristic which does not have associated with it an extremely low radiation resistance. As the antenna height is increased the radiation resistance in general will increase to reasonable values and hence give good antenna efficiency.

The power rating for loaded antennas should not include the conductor resistance losses of the loading system. On this basis even low efficiency loaded antenna systems can be made to increase the primary coverage of a radio broadcast transmitter, while at the same time reduce fading and distant interference with other stations.

The author believes that further work should be carried on along the lines of an investigation of the radiation properties of solid contour surfaces, if extreme heights are to be avoided. Since air nagivation limits

the height of most broadcast antennas, this seems to be the logical road to further improvement. Similitude measurements on models at short wave lengths should be a good method to use in such an investigation.

ACKNOWLEDGMENT

In conclusion it is the writer's pleasant duty to thank Mr. E. L Gove, technical director of the Radio Air Service Corporation, for his assistance and suggestions in the directing of this investigation. Also, for the help rendered by Messrs. J. C. McNary and Karl Spangenberg, Prof. W. L. Everitt, Dr. R. S. Burrington and the radio operators of radio station WHK.

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THE MEASUREMENT OF RADIO-FREQUENCY POWER*

By
A. Hoyt Taylor

Summary—*The possibilities of the cathode-ray wattmeter have been investigated between sixty cycles and forty megacycles. Its use can be extended to between 100 and 200 kilocycles by the use of either selective or nonselective amplifiers, thus permitting a wide range of power sensitivity. When the nonselective amplifier is used or when no amplifiers at all are used, the power measured includes all the harmonics; when the selective amplifier is used, the power is limited to the power on that frequency to which the amplifiers are tuned. The wattmeter has been extended by the use of a suitable pickup mechanism and suitable selective amplifiers to forty megacycles. For some purposes it is an indispensable instrument and for others a very useful instrument permitting more rapid, if not as accurate, measurements than are possible with other methods.*

Most of the well-known methods for the determination of radio-frequency power are subject to certain errors and certain limitations of application. One very well-known method often used for the determination of the power output of transmitters involves their operation into a dummy antenna whose effective resistance is known. The measurement of power is accomplished by multiplying the known resistance by the square of the observed radio-frequency current. At relatively low frequencies this method is satisfactory for many purposes. It is safe enough at low frequencies to assume that the current in one part of the resistor is the same as in all other parts; it is easy to provide satisfactory resistors for almost any reasonable amount of power, and it is not difficult to make what small corrections may be needed for skin effect. Finally, it is possible, by using properly calibrated ammeters adjusted sharply to respond only at definite frequencies, to break down the total power into the fundamental and harmonic components. However, as we proceed to higher frequencies all of these things become more difficult. It is no longer safe to assume a uniform distribution of current in the load resistor; it is not always easy to allow for skin effect, and it is often very difficult to provide any suitable resistors adequate for dissipating the necessary amount of power. Moreover, the resistors are likely to be inductive.

One common method of avoiding some of the difficulties set forth at the conclusion of the preceding paragraph is to operate the source of power into a lamp load. The purpose of this sort of operation is to pro-

* Decimal classification: R240. Original manuscript received by the Institute, May 6, 1936. Presented before joint meeting U.R.S.I.-I.R.E., Washington, D. C., May 1, 1936.

vide a load of high specific resistance and very small diameter of conductor, thus reducing skin effect. However, in order to avoid any ambiguity as to just what power is consumed in the load, it is customary to compare the brilliancy of the lamp under the influence of its high-frequency load with its brilliancy when excited by sixty cycles or direct current. However, conditions can readily arise where this procedure is not satisfactory. In order to measure considerable amounts of power, it is often necessary to build the filaments in spirals of small diameter. This immediately gives a considerable reactance component to the load and permits the establishment of standing waves, especially if harmonics are present. One instance is recalled where such a lamp operating at fourteen megacycles developed a rather strong forty-two-megacycle harmonic which happened to form a standing wave on the filament so that the filament was very much hotter in some places than in others. There is a double disadvantage to this; first, even though our brilliancy measurements may lead us to a fairly correct knowledge of the lamp resistance at the fundamental, we cannot safely assume that the lamp has the same resistance for harmonic currents; furthermore, our brilliancy measurements are not worth much because we do not know what part of the filament to look at, and it is difficult to get an average brilliancy and interpret its physical meaning. Even if we break down the current into the harmonic components by the use of uniwave ammeters, we cannot derive from this information the distribution of harmonic power because the load has resistances differing for fundamental and for each harmonic. Finally, at very high frequencies it is almost impossible even in the absence of harmonics to get an even temperature on the lamp filaments from one end to the other. The lamp load method certainly has its conveniences and useful applications, but it is capable of giving errors in excess of twenty-five per cent in the power determined if conditions happen to be particularly unfavorable.

When we come to the measurement of the power from a transmitter into an actual antenna, we find ourselves still more limited in the number of available methods for power determination and in the accuracy of those that we do use. There are certain well-known forms of antenna which, if erected in a free space, would have, looked at from some particular point where we might install an ammeter, a known and calculable resistance. Actually, however, antennas never function in free space but are much influenced, especially in their effective resistance, by their immediate surroundings. Then, too, it is often necessary, especially in the naval service, to operate an antenna, at many frequencies, and it is difficult to determine accurately the effective resistance at all these different frequencies, for many of which the antenna will

not be in a resonant condition. It should also be noted that the performance of the transmitter into a dummy antenna may not be the same as when operating into the real antenna for which it may be designed and there is a very bad discrepancy if it is forced to operate into an antenna for which it is not specifically designed. The greatest difficulty comes in the distribution of harmonic current and harmonic power. A transmitter which might be expected to perform fairly well as far as harmonics are concerned on a suitable dummy antenna, can show a much higher harmonic content with the antenna into which it must ultimately operate.

The considerations mentioned proved an incentive to investigate the possibilities of less familiar methods of measuring alternating-current power. Since it is very natural to correlate the measurement of en-

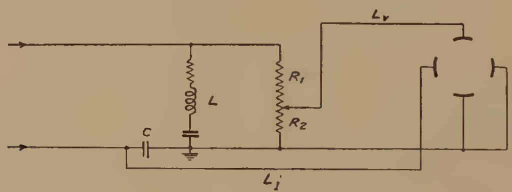

Fig. 1

ergy or of power with the measurement of an area, it is very natural that the cathode-ray oscillograph should have been used early for this purpose both in this country and abroad. However, a search of the literature failed to disclose any serious attempts to make practical application of the cathode-ray oscillograph for this purpose except at relatively low frequencies. Apparently no determined attempt had been made to see what frequency limits could be reached and how practical an instrument could be produced. It was decided to start the investigation at sixty cycles and work upward. Fig. 1 shows a very useful circuit at low and moderate frequencies, say twenty-five cycles to 100 kilocycles. The load L consisting of resistance, inductance, and capacitance, is supplied power from the lines entering at the left of the diagram, provision being made for grounding one side of the circuit if so desired. In series with the load is a capacitance of suitable size, C. The voltage drop over this capacitance is led to one pair of plates of the oscillograph. Across the load is placed a very high resistance, nonreactive potentiometer made up of the resistors, R_1 and R_2. The drop over the smaller of these two resistors is then led to the other pair of plates on the oscillograph. In the following discussion

A = area traced out on the oscillograph screen.

x = linear deflection along the voltage axis.

y = linear deflection along the current axis.
q = instantaneous value of charge on condenser C.
K_1 and K_2 are constants of proportionality depending on sensitivity of the oscillograph.
T = period of a complete cycle.
F = frequency of a complete cycle.
e and i are instantaneous values, respectively, of voltage and current in the load.

$$A = \int x dy : x = K_1 e : y = K_2 \frac{q}{C}$$

$$\frac{dy}{dt} = \frac{K_2}{C} \frac{dq}{dt} = \frac{K_2}{C} i \qquad dy = \frac{K_2}{C} idt$$

$$dA = K_1 e \frac{K_2}{C} idt$$

$$A/\text{cycle} = \frac{K_1 K_2}{C} \int_0^T eidt$$

$$\text{power} = \frac{1}{T} \int_0^T eidt = F \int_0^T eidt$$

$$\text{power} = \frac{F\ A/\text{cycle}\ C}{K_1 K_2} = KFCA.$$

In the discussion it is not assumed that the current in the load L is sinusoidal in character but it is assumed that it is a periodic function. It will be seen that at any given frequency there is a linear relationship between the area of the figure on the oscillograph and the power and that this is independent of the wave form; in other words, the power represented by the area includes the harmonic power. Since we shall want from forty to 100 volts on the terminals of the oscillograph in order to insure adequate sized areas, the method shown in Fig. 1 will not operate at low power. There is no particular upper limit of power, however, since high voltages can be reduced by a suitable potential divider and high currents can be accommodated by enlarging the value of the capacitance C. If transmission lines are used to connect the load with the oscillograph proper, they must be of substantially the same length for both the current legs; otherwise, a phase-adjusting mechanism will have to be provided somewhere. It is customary to check phase by substituting for the load L a pure capacitance of suitable value. The trace on the oscillograph should then be a straight line but not neces-

sarily at 45°. The slope of the line will depend upon the choice of R_2 and capacitance of C. If the pure capacitance shows a thin small slanted ellipse it means that there is a phase error somewhere, probably in the resistors R_1 and R_2 which have to be chosen with great care as the frequency is raised. In order to make the method available for measurements at low power, it is desirable to include amplifiers in both voltage and current legs. However, these amplifiers must be designed with great care; they must be flat over the range of frequencies to be measured, including all probable harmonics, thereof, and they must have either no phase distortion or if there be a small phase distortion, it should be the same for each amplifier. Since this condition is sometimes difficult to realize perfectly it is advisable to have a resistance capacity combination between the output of one of the amplifiers and the oscillograph in order to make small angular shifts of phase, thus enabling a perfect line-up on the phase when the test capacity is put in the place of the load. The first radio-frequency wattmeter constructed at this laboratory was intended only to cover from twenty-five cycles to 100 kilocycles and it was built along the lines just described. The oscillograph was mounted with the tube vertical, the screen end level with the top of a "tea wagon." The amplifiers, standard calibrating resistances, potentiometer, etc., were all mounted in one compact portable assembly. The instrument was calibrated by applying a reasonably pure sine wave to a standard noninductive resistor placed in the load position. Thus, the power in the load can be calibrated as i^2r and related to a measured area. By carrying out this calibrating at a number of different levels a check may be had of the linearity of the system. Needless to say, any lack of linear response on the part of the amplifiers or oscillograph introduces errors and it is very difficult to devise any means of calibrating to allow for that. If all measured areas were of the same shape, due allowance could be made for lack of linearity but since some areas are nearly round and others are slender ellipses sliding over into either the inductive or capacitive quadrants, the effect of nonlinearity is decidedly complicated. If we would presuppose a knowledge of the phase angle and full knowledge of the response curve of both amplifiers, we could make a correction. But since one of the prime purposes of such a measurement is to determine the phase angle, this sort of thing would be useless; when we consider further the presence of harmonics, the picture becomes still more complicated. The only satisfactory solution is to have amplifiers reasonably good over the range of frequencies and voltage to be handled and be satisfied with the resulting accuracies.

This first instrument having proved very useful for a variety of pur-

poses, it was decided to extend the measurements to higher frequencies. The circuit shown in Fig. 1 is capable of operating up to several hundred kilocycles without serious difficulty as long as one does not use amplifiers, i.e. as long as there is sufficient power available to measure, but to obtain a wide range of sensitivity it is absolutely necessary to use amplifiers. Some amplifiers used in television work might be useful at this stage of the investigation but it is unlikely that one having phase fidelity, linearity, and reasonably flat response could be expected to go higher than 500 kilocycles. Since it was desired to push the investigation into the superfrequency band, the decision was made to go to the tuned radio-frequency type of amplifier. Once this decision was made there was no longer any particular reason to continue the use of the condenser pick-off at C in the current leg or the use of the potentiometer R_1 and R_2. Experiments showed that a current transformer at position C was equally useful since the use of the sharply tuned radio-frequency amplifiers limits the measurement to a particular frequency and excludes harmonics. Moreover, many situations have arisen where it is highly desirable to put the current transformers in the high potential side of the load rather than in the low potential; still further situations would require push-pull operation as in transmission lines and, therefore, it was decided to build two current transformers whose secondaries would pick up a voltage proportional to the resultant current; one such could be placed in either side of the transmission line. For the voltage pick-off it was decided to use a small capacitive coupling. This also was arranged in duplicate so that push-pull operation would be possible. Considerable study had to be made of the current transformer if it was to be used at a high potential point since the situation was very complicated if both electrical and mechanical couplings were allowed to function simultaneously between the primary and secondary of the transformer. The current transformer was ultimately made up with a single turn primary about two inches in diameter and a multiple secondary variable from three to six turns, electrically shielded from the primary.

Figure 2 gives the schematic arrangement of parts for this assembly. The load is represented as a combination of resistance, inductance, and capacitance at L, the power supplied from the lines entering from the left. The two current transformers are labeled CT_1 and CT_2, respectively. An electrical shield made of fine copper wire open at one end and grounded at the other is placed between the primary and secondary. The secondaries of these two transformers are arranged in series and connected to the transmission line. The voltage delivered to the transmission line is therefore proportional to the vector sum of the

high-frequency currents through the primaries of the transformers CT_1 and CT_2. The right-hand end of this transmission line is connected to the input of the radio-frequency amplifier A_2. The input coupling coil is electrically shielded from the grid coil of the amplifier to which it is coupled. A similar transmission line leads from a similar amplifier A_1 to the voltage pick-off which is connected to the terminals of the load by way of the two pick-off condensers C_1 and C_2. These condensers are so constructed that they are thoroughly shielded and adjustable over a considerable range of capacitance. However, this capacitance is always kept small. The condenser plates are two disks two and one-half inches in diameter. The two current transformers are also care-

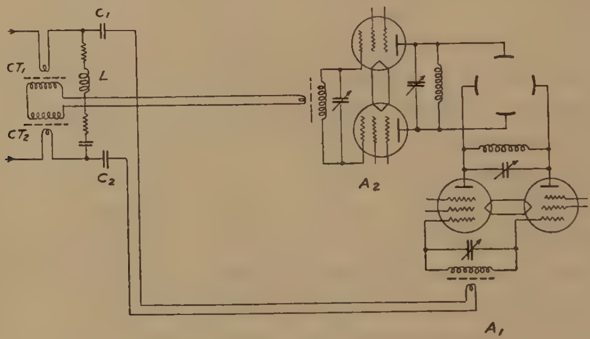

Fig. 2

fully shielded. Provision is also made for external shunts on the primaries of these transformers, these shunts being necessary when measuring heavy currents. The apparatus to the left in Fig. 2 is mounted in one unit separate from the wattmeter proper, this unit being so arranged that it can be placed at the side of a transmitter near the output terminal or near the input or output of transmission lines. It is called the pickup mechanism. The transmission line may be of any reasonable length, it being advisable in spite of careful shielding to remove the amplifiers and associated oscillograph some distance when the output of a powerful transmitter is being measured. We have found the RCA 905 to be a satisfactory cathode-ray tube for this purpose and the RK 25 tubes make reasonably satisfactory amplifiers. These amplifiers are provided with plug-in coils and two sets of input and output tuning condensers so that the total frequency range covered is from 200 kilocycles to forty megacycles. It is advisable to mount the cathode-ray tube in such a way that the leads are as short and as symmetrical as possible. Otherwise the output LC ratio of the amplifiers will be too low, cross-coupling will exist and the apparatus will function

badly at the higher frequencies. For transmission lines it is advisable, on account of certain difficulties to be mentioned later, to use lines of reasonably low attenuation and a very low impedance sheath. Fig. 2 shows a symmetrical circuit such as would be used with the output of a push-pull driver or transmitter or in working with a symmetrical transmission line. However, if a unilateral circuit is used, one of the current transformers, as CT_2 for instance, can be omitted and one side of the voltage line can be grounded as at C_2. There is an alternative method of connecting the voltage pick-off leads; viz., ahead of the current transformers instead of following them. In the method illustrated in Fig. 2 any losses in the voltage measuring circuit will be reflected as an error in the wattmeter reading. If the alternative method of connection is used, any losses in the current transformers will be reflected as an error in the reading. Fairly satisfactory measurements have been obtained between 200 and 30,000 kilocycles. Between 30,000 and 40,000 kilocycles discrepancies and errors rapidly increase so that at 40,000 kilocycles the accuracy is probably not better than fifteen per cent. The apparatus is sufficiently flexible to cover a wide range of power down to 1/100 of a watt and also to cover a wide range of both current and voltage.

Method of Calibration

Since changes in phase occur not only in the pickup mechanism but in the amplifiers themselves according to the tuning thereof, it is necessary after adjusting the instrument to the proper sensitivity level for the currents and voltages to be measured, to obtain a preliminary set of phase. The supply voltages to the power packs, not shown in Fig. 2, which feed the oscillograph and the amplifiers, are run through voltage regulators. It is advisable to turn on these power packs and the amplifiers sometimes before measurements begin in order that temperature changes may not cause shifting in phase. However, this is not as important as might be supposed because the amplifiers reach equilibrium within a short time and whatever phase shifts there may be are approximately symmetrical in the two amplifiers and therefore the differential phase shift is very small. There will, however, be some small change in sensitivity due to temperature changes if the preliminary warming-up process is not used. Even when operating as high as thirty megacycles it was found that the settings could be readily duplicated from day to day. The phase adjustment is made as in the case of the instrument illustrated by Fig. 1; viz., by putting an Isolantite insulated capacitance of suitable dimension in place of the load. This has a negligible power factor in so far as its bearing on the accuracy of

these measurements is concerned. Assuming this power factor to be zero, the figure on the oscillograph is then adjusted by slightly mistuning one or both of the tuning controls on one or the other of the amplifiers. Having set the phase properly, it is then necessary to calibrate the wattmeter on a standard load. This is a matter of great difficulty for high power loads and has been superseded by another method which will be described later. For light loads up to twenty-five watts, we have found the so-called Zircon resistors most satisfactory. While these resistors have some temperature coefficient they do not heat up unless left in the circuit for long periods, and calculations have shown that their skin effect should be almost negligible even at 100 megacycles. Because they may vary from day to day, their direct-current resistances are frequently measured. On the whole the shifts have not been large and they may be considered one of the most satisfactory resistances available for very high-frequency work. Before accepting these resistors they were subjected to rigid tests. Suitable drivers of pure wave form were constructed for supplying the load and covering a wide range of frequencies. Certain lamps of simple structure and calculable skin effect were then constructed and used up to fifteen megacycles for calibrating the wattmeter. The lamps themselves were, of course, checked by observing the brilliancy under equivalent sixty-cycle loads, using for this purpose the Weston Photronic cell. A number of Zircon resistors of various sizes were then measured, using the wattmeter calibration as given by the standard lamps. The agreement between the high-frequency and the direct-current resistance was so good that it was evident that the departure was small enough to fall within the limit of accuracy of our measurements. Small errors indeed were discovered at first, but they were traced to errors in the radio-frequency meters used to determine the calibration current. Therefore, all radio-frequency meters were subjected to a calibration at radio frequency before being used in this work. These meter errors are in many cases so large that they warrant a special investigation which it is hoped will be subsequently reported in another paper. Suffice to say that means to allow for these errors were worked out. Having found by theory and experiment that the Zircon resistors had no measurable skin effect at fifteen megacycles, it was assumed safe to consider that they would be satisfactory up to forty megacycles since theoretically they should be satisfactory up to 100 megacycles. In connection with standard resistors, another type was worked out which was made up of five parallel connected windings of No. 44 Cupron wire, each layer being wound on a small mica strip, each strip separated by a small air space from its neighbor and all mounted on a General Radio plug. These resistors will carry 500 milliamperes for short periods and 300 continu-

ously. They have very little temperature coefficient and have no serious inductive reactance up to fifteen megacycles. At thirty megacycles their inductive reactance is entirely too high. These resistors were made up in units from one to twenty ohms and could be combined usefully in various ways. Incidentally, as checked against the standard lamp and against the Zircon, their resistance up to fifteen megacycles was sufficiently close to the direct-current value so that no correction was needed. A number of other alloy wires were tried but with no satisfaction. In most cases the inductive reactance was very high and the departure from the direct-current resistance value was very great. Below five megacycles it was found that the Ward Leonard plaque resistors, both the two-ohm and the twenty-five-ohm sizes, were very good and very convenient, especially as they will carry more power than either of the other two resistors. Beyond five megacycles their inductive reactance is very high, although their resistance still does not depart much from the direct-current value. Inductive reactance is very objectionable in resistors at very high frequencies as in a given resistor assembly it is so easy under these conditions for standing waves to develop. Excessive voltages are required to overcome the inductive reactance and this all results in an elongated narrow ellipse in the inductive quadrant on the oscillograph which is not easy to measure accurately and is subject to large errors if there is a very small mistake in the phase-angle setting. This method of calibration then consisted in inserting a standard resistor of appropriate value, measuring the current through it with a previously calibrated radio-frequency meter, and observing the area produced by the oscillograph.

In order to lead up to the second method of calibration, which is especially suitable for high power, it is advisable to give a simple analysis of the circuit of Fig. 2. The symbols bear the same meaning as in the previous analysis of Fig. 1 except that K_1 and K_2 now include components introduced by the amplifiers A_1 and A_2 and therefore are functions of the frequency. They also include the transfers through the voltage pick-off condensers and through the current transformers which are again functions of the frequency. One new symbol is introduced, δ, which is the arbitrary shift of one of the amplifiers away from resonance, which shift is necessary to line up the phase.

$$x = K_1E \sin \omega t$$

$$y = K_2I \sin (\omega t + \phi + \delta)$$

$$A = \int xdy$$

$$= K_2K_1EI\omega \int_{\omega t=0}^{\omega t=2\pi} \sin \omega t \cdot \cos (\omega t + \phi + \delta) d\left(\frac{\omega t}{\omega}\right).$$

Let,

$$\alpha = \phi + \delta$$

$$\cos(\omega t + \alpha) = \cos \omega t \cos \alpha - \sin \omega t \sin \alpha$$

$$A = K_1K_2E \frac{I\omega}{\omega} \int_0^{2\pi} (\sin \omega t \cos \omega t \cos \alpha - \sin^2 \omega t \sin \alpha) d(\omega t)$$

$$= -2K_1K_2 \frac{EI}{2} \pi \sin(\phi + \delta)$$

$$= -2K_1K_2 \frac{EI}{2} \pi \cos\left(\frac{\pi}{2} - \phi - \delta\right)$$

$$\text{if} \quad \frac{\pi}{2} - \delta = 0 \text{ by adjustment}$$

$$A_{\text{cycle}} = 2\pi K_1K_2 \frac{EI}{2} \cos \phi$$

$$\text{power} = \frac{A_{\text{cycle}}}{2\pi K_1K_2}.$$

If the phase angle is zero, i.e., the load is wholly resistive, the above equation can be derived very simply by computing the area of the ellipse which will have a semidiameter one way of K_1E and a semidiameter the other way of K_2E. Hence,

$$A = \pi K_1K_2EI = 2\pi K_1K_2P.$$

From the above equation it is evident that if one knows the sensitivity of the current leg in centimeters deflection per ampere and the sensitivity of the voltage leg in centimeters deflection per volt, one can compute the calibration constant or watts per unit area. Now, if we have a driver of pure wave form and a properly calibrated radio-frequency ammeter, we can get both these quantities without recourse to high power. The current leg can and should be calibrated practically on a short circuit, that is to say, at low voltage, and the voltage leg can be independently calibrated by the same method using the potential drop over a very high resistance with a small current through it. The drop over a known capacitance can also be used for this calibration and voltage but it is difficult because at high frequencies the inductance of the leads, short as they may be, will introduce some error. This obviates a very serious limitation on the use of the instrument. At very high frequencies there has so far not been produced a satisfactory standard resistance capable of dissipating high power.

Discussion of Errors

(a) *Linearity*

The error introduced by improper setting of phase and by improperly calibrated radio-frequency ammeters have already been discussed. The choice of the proper voltages for the different elements of the tubes should be so made that the system as a whole will be as nearly linear as possible. This includes, of course, the pickup device, the transmission lines, the amplifiers themselves, and the oscillograph. With a fixed frequency (obviously this should be held constant during calibration and measurement) there is no reason to suspect nonlinear performance in either the pickup device or the transmission line. It is possible to find a certain degree of nonlinearity in the cathode-ray tube and in the amplifiers. In testing for nonlinearity the procedure has been to test each leg independently from the pickup device clear through to the oscillograph and to adjust the voltages on the amplifier elements for the most linear over-all response. No question of harmonics enters in here because the two tuned circuits in the amplifiers effectively exclude them. It has been possible to make this device sufficiently linear over all to make very practical measurements up to thirty megacycles and useful measurements somewhat beyond, how far we do not yet know.

(b) *Effect of Modulation*

The effect of frequency modulation is disastrous as the nature of the amplifiers is such that rapid changes in both phase and amplitude will occur as the frequency varies. The effect of energy modulation is to produce the well known cat's-eye figure which is an ellipse within an ellipse, the intervening space between the two being filled with suffused light. These areas represent minimum and maximum powers and may be readily used if so desired for a determination of the percentage of modulation. However, if the modulation introduces radio-frequency harmonic distortion, the power involved therein will not be measured but only the fundamental power.

(c) *Effective Shielding*

Very effective shielding and by-passing must be used so that there will be no intercoupling between the two amplifiers by way of power supply and so there will be no direct pickup between the amplifiers and the power source except what is delivered over the transmission lines. Intercoupling between the amplifiers is ruinous for many reasons but particularly because it is liable to lead to regeneration involving rapid phase shifts and nonlinearity. Tests for interaction between the ampli-

fiers may readily be made by tuning both of them to the frequency in question and exciting one only from the transmission lines. If the other amplifier also shows a considerable deflection there is interaction, indicating improper by-passing of leads to the common power pack or incomplete shielding. Pickup between the amplifiers and the source of power may, of course, be tested by disconnecting both inputs.

(*d*) *Phase Displacement Due to Time of Flight of Electrons in Tube*

There is a phase angle introduced into the figure on the oscillograph due to the time of flight of the electrons from under one pair of deflector plates to under the other pair. At ten megacycles this may amount to eight or ten degrees and will have higher values on higher frequencies. This is automatically taken up in the initial phase-setting operation since we adjust the figure on the capacitive load to a straight line.

(*e*) *Cross Coupling*

This name has been given to a possible unbalanced coupling which usually exists in most cathode-ray tubes between the two pairs of elements. This may be within the tube itself or may involve the arrangement of the leads. It can be neutralized by a very small capacity usually not over two or three micromicrofarads connected between the proper pair of electrodes so as to balance this effect out. Having first ascertained that there is no reaction between the two amplifiers, it is only necessary to place them both in tune, exciting one of them and then adjust the small condenser until the very small deflection on the opposite axis disappears. A false adjustment can, of course, be made if there is interaction between the amplifiers but this state of affairs will betray itself by dependence of the phase setting on the amplitude because there will be regeneration present in nearly every case. Therefore, after balancing out the cross coupling, it is well to put in the phase-setting condenser, recheck the phase, and observe that the slope of the line does not change when the current through the phase-setting condenser is increased and diminished. The line should change its length but not its slope. Neither should it open up into an ellipse. If it changes its slope there is a different degree of linearity on the two axes; if it changes into an area (small ellipse) there is probably regeneration and interaction between the amplifiers. Good design will eliminate these difficulties.

(*f*) *The Effect of Radio-Frequency Ground*

As in most measurements of high frequency the question of suitable ground connections is a very important one. Theoretically there should

be a perfect ground connection at the pickup mechanism and its associated shields and at the amplifier with its associated shields. Practically, the use of leads connecting to water pipes, steam pipes, etc., is useless because these leads cannot be made short enough to have a low reactance and the result will be that the pickup system as a whole will be oscillating above and below ground potential by an amount different from that at the wattmeter proper, including the amplifier. Therefore, a counterpoise ground is very much to be preferred. Such a ground may have strips of thin copper or copper screening about one foot wide which have been found useful. Moreover, it is advisable to connect the counterpoise system at the pickup mechanism with that at the wattmeter proper with additional strips of the same material, upon which lie the transmission lines connecting these two parts of the mechanism. Finally, in order to avoid accidental pickup of voltages by the transmission line from sources not involved in the measurement, it is highly necessary that the sheaths of the transmission lines be grounded at numerous points to the counterpoise system. To test for this type of stray pickup it is advisable to note the deflections on both axes of the oscillograph under the following conditions: First, when the connecting plugs at the end of the transmission line are disconnected from the pickup mechanism; second, when they are heavily short-circuited; and third, when the pickup mechanism itself is entirely disconnected from the transmitter but the transmitter itself is in operation. Naturally, this sort of stray pickup increases as the frequency is increased and must be carefully guarded against principally by proper choice of counterpoise grounds. These grounds must be brought clear up to the device in question and connected with extremely short and heavy leads of low impedance.

(*g*) *Method of Measurement of Area*

It is possible with one type of planimeter with a separate table for the rolling wheel to measure directly on the end of the oscillograph tube. However, this introduces an error on account of the curvature of the tube, and it has been found on the whole much better to make a copy of the trace on thin paper and subsequently to measure this trace with the planimeter. After a little experience the operator becomes extraordinarily accurate in this work and there is little difficulty in repeating planimeter measurements made this way to better than one per cent unless the areas are abnormally small. The misbehavior of the planimeter when operating directly on the end of the cathode-ray tube could be corrected if the tubes had a flat end but this would weaken the structure of the tube and would, moreover, introduce another error in the nature of a nonlinearity of the tube deflections themselves.

(*h*) *Transmission Lines*

Some work was done at thirty megacycles with transmission lines having rather high impedance sheaths. This resulted in undesirable stray pickup and other difficulties which were effectively eliminated by using properly constructed lines. For a nonportable setup, concentric transmission lines with copper tubing and Isolantite beads would certainly be preferable. A transmission line with a poor sheath can even be used if the sheath is not covered with insulation and if that sheath is grounded frequently to the counterpoise system. This prevents large differences of potential existing in the sheath. It should be mentioned that these difficulties are all a great deal less troublesome with push-pull operation and symmetrical circuits. They are principally troublesome with a unilateral circuit, one side of which has to be grounded.

(*i*) *Effect of Screen Current in the Current Transformer*

Since it is necessary in many cases to have the current transformer on the high side of the load, the question arises to what effect is produced by the capacitive current between the primary of the transformer and the electrical screen between the primary and the secondary. At high frequencies this current may be of sizable dimensions. A great many tests have been made as to the effect of this screen current and it has been concluded that with properly designed transformers, this current is practically wattless and therefore does not influence the area which determines the power. It does, however, modify the shape of that area since the capacitance between the primary of the current transformer and the screen is in effect a capacitance in parallel with the load. The only place where the screen current has proved to be embarrassing is in the determination of the current constant of the instrument which is used in calculating its sensitivity for high power. Whenever this current constant is determined it should be determined at low power, although with the necessary current commensurate with the measurements to be made, and the current transformer should be as near ground potential as is possible. Under these conditions the screen current is negligible. Since both methods of calibration are available at low power, say under twenty-five watts, it is possible to check the validity of this reasoning by determining the constant by both methods and comparing the results; i.e., the constant is computed from the volt sensitivity and the current sensitivity and then it is also directly measured by getting the area for a known current through a standard resistor. Since even as high as at thirty megacycles these results usually agree within five per cent it is concluded that the screen current does not introduce any very serious losses.

(*j*) *Losses in the Current Leg*

Any method of measurement requires the consumption of some power. If the amplifiers are sufficiently sensitive and the transmission lines are not too long and are operated close enough to the matching point, these losses can be kept low. It is evident from an inspection of the circuit in Fig. 2 that the larger losses are to be expected in the current transformer. Fortunately, there is a way of determining this if we use the alternative connection of the voltage leg; viz., ahead of the voltage transformer. In this case as in any voltmeter and ammeter method, the losses in the voltmeter are out of the picture and only the losses in the ammeter are involved. If, then, we determine a series of areas on a number of different standard resistors all carrying the same current and plot these areas versus resistance in a curve, we shall determine two things: First, if the curve is a straight line as it should be, the instrument is giving a linear response. If this is not true, the voltages on the amplifier should be adjusted to the best possible approximation to over-all linearity; second, if there are losses in the current leg, then this curve when extended will not pass through the origin of co-ordinates but will intersect the resistance axis of the curve at a point to the left of zero, thus indicating the equivalent resistance of the current pickup. When working at very small powers, say of the order of one-tenth of a watt or less, and at frequencies as high as thirty megacycles, the current pickup may have an effective resistance as high as twelve ohms, but when working at higher power levels the coupling between primary and secondary of the current transformer is reduced and an external shunt is usually added on the primary. Even at high frequencies, this usually reduces the loss resistance of the current pickup to less than an ohm.

(*k*) *Loss in Voltage Pickup*

The loss in the voltage pickup, while unquestionably much smaller than in the current pickup, is harder to determine. The method of connection of the voltage leg as shown in Fig. 2 is then used and since the simplest way to consider this loss is as a high shunt resistance across the load, it can be determined by carrying out a series of measurements at constant voltage and a variety of currents with a series of resistors. The curve which we now plot will be reciprocal resistance against area. However, this method is not satisfactory at high frequencies where we really need it. The losses are known to be too small to bother with at the lower frequencies. It fails at high frequencies because of the uncertainty as to the voltage, and this uncertainty comes about because of the inevitable inductive impedance of the leads used with the various

resistors. True, this can all be allowed for or separate voltage indicating instruments can be used, but it is a laborious process. Therefore, the alternative method of connection whereby the voltage leg losses are excluded from the picture is usually to be preferred. However, a check on the magnitude of the voltage leg losses can easily be had by measuring a load by the method of connection shown in Fig. 2 and correcting for the losses in the current transformer; then measuring again by the alternative method of connection whereby the voltage pick-off is placed ahead of the current transformer, and comparing results. If there is a considerable discrepancy it must be due to the losses in the voltage leg. Usually the discrepancy is well within the limits of error of the measurement. This assumes, of course, reasonably sensitive amplifiers in the voltage leg.

(*l*) *Reliability of Measurements*

It is believed that if precautions outlined above have been taken, power measurements can be relied upon to within seven per cent at thirty megacycles and to within three per cent at 200 kilocycles.

Applications of the Cathode-Ray Wattmeter

(*a*) *Measurement of Resistance, Reactance, and Power Factor*

The measurement of resistance as already indicated is carried out by determining the power in the load with the wattmeter and dividing

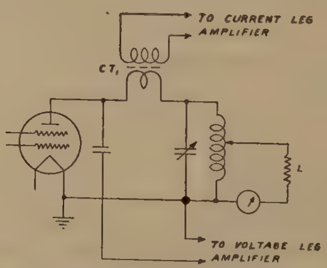

Fig. 3—Measurement of output of radio-frequency amplifier.

by the square of the current as measured by a suitably calibrated ammeter. If the voltage and current axes have been independently calibrated for current and voltage sensitivity, the impedance measurements may be made in the usual way by taking the ratio of the volts as indicated on one axis of deflection and the amperes as indicated on the opposite axis. The power factor can be determined by measuring the area of the power ellipse and then computing the area of another ellipse whose axes are parallel to x and y and whose diameters would be

the maximum x deflection and the maximum y deflection, respectively. This theoretical ellipse represents the apparent power of the circuit and the power factor is the ratio between the area of the measured ellipse and the area of this calculated ellipse. The reactance may be determined if we have first determined the resistance and power factor. The direction in which the ellipse slants will indicate whether the reactance is inductive or capacitive; i.e., if we have made our phase set so that the straight line obtained at that time lies in the first and third quadrants, these will be the capacitive quadrants and the second and fourth quadrants will be the inductive quadrants.

(b) Antenna Characteristics

The power, phase angle, resistance, and impedance of an antenna may be determined with the wattmeter at any frequency within its range and for any kind of an antenna, tuned or untuned. This is one of the most valuable applications of the wattmeter because it is very difficult to get, by any other method, these constants for an antenna which has traveling waves rather than standing waves. The current

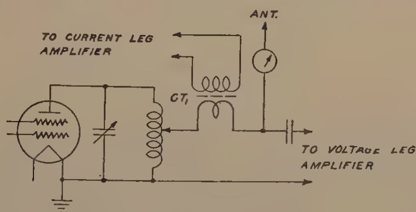

Fig. 4—Measurement of antenna power, fundamental or harmonic.

transformer for an antenna operated against ground is placed directly in series with the antenna which is coupled in any of the standard ways with the transmitter or driver, and the voltage pick-off is connected by either of the methods mentioned earlier in this paper (see Fig. 4). The component parts of the impedance are determined as indicated in (*a*) above. For symmetrical antennas such as doublets, the push-pull connection is used and the use of the wattmeter is particularly valuable here because if there be any asymmetry in the system, it will nevertheless be taken care of because the two current transformers give a resultant pickup proportional to the vector sum of the currents in the two legs.

(c) Properties of Transmission Lines

The wattmeter is very useful connected to either input or output of a transmission line. If connected to the input we may assume an

arrangement similar to Fig. 2 except in the place of the load L we connect on the two transmission lines. The correct match in the far end of the line may be approximately determined (for a reasonably good line at least) by adjusting the coupling to the load at the far end until the axes of the power ellipse are closely parallel to x and y, respectively, indicating 100 per cent power factor for the system. If the known resistance load be substituted at the far end of the line, preferably in the form of two equal resistors with a calibrated ammeter between them, the attenuation of the line may be determined by comparing the output power computed from I^2R and the input power as indicated by the ellipse on the oscillograph. It is possible to make measurements very rapidly by this method.

(*d*) *Output of a Transmitter*

As already indicated, the output of a transmitter can readily be measured for either unilateral or push-pull types by connecting the current transformers directly in series with either an artificial or real antenna. It should be remembered, however, that the power measured is the fundamental power only, to which the wattmeter is tuned.

(*e*) *Radio-Frequency Efficiency of Tubes* (*See Fig. 3*)

Studies are now being carried out on the radio-frequency conversion efficiency of tubes used as power amplifiers. It is necessary to have a driver of requisite power for calibrating purposes and of pure wave form. The same driver is used for exciting the grid of the tube under study. The current transformer is connected directly between the plate terminals of the tube and the tuned plate tank circuit. For push-pull tubes both current transformers are used. The voltage pick-off is connected in the usual manner which means for a single tube between one terminal of the current transformer and ground and for push-pull operation the two points of connection are two corresponding terminals of the current transformer. However, there is a limitation as to frequency in this method of determining the total radio-frequency power. We are, of course, measuring the radio-frequency component of plate current and the associated radio-frequency plate voltage. The power measurement involves the power in the tank circuit plus the power in whatever load is coupled to that tank. Now, at low frequencies, assuming the plate tank circuit to be in tune, a small current will flow at quite a large voltage. The range of sensitivity on both current and voltage legs of the wattmeter is adequate to take care of the situation. Moreover, since under these conditions we shall have a good power factor, we shall get an ellipse of such dimensions as to permit accurate

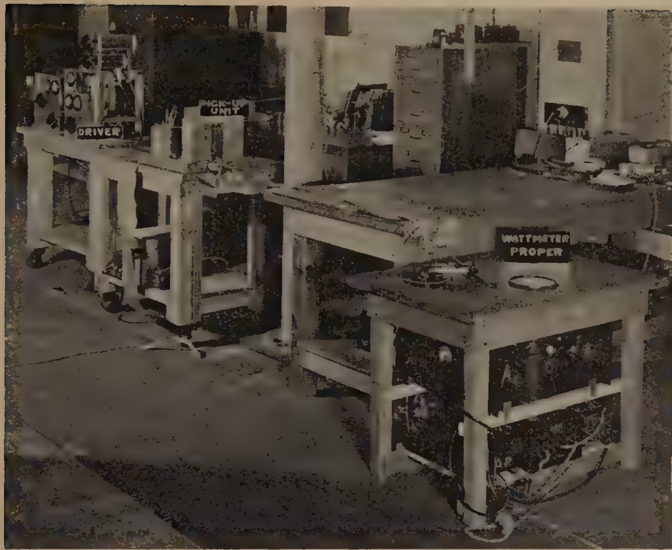

Fig. 5—Arrangement for typical tests showing driver, pickup unit, and wattmeter proper.

Fig. 6—General view of wattmeter proper showing voltage amplifier A_e, oscillograph O, current amplifier A_i, amplifier power pack $A.P.P.$, oscillograph power pack $O.P.P.$

measurement. At very high frequencies, however, the plate capacitance acts as a considerable fraction of the total tuning capacitance of the plate circuit and since we are measuring only the external circuits, the impedance of the external circuit at high frequencies will inevitably be highly inductive. This leads to a long narrow ellipse slanted over into the inductive quadrant. Such an ellipse cannot be measured as accurately as the shorter but broader ellipses one obtains at lower frequencies. The effect of a small error in phase set is very much larger. The work being done at present on this part of the problem is being carried out between five and ten megacycles. Having determined the total radio-frequency power in the tank circuit plus load, the wattmeter can then be connected in the load alone whether it be artificial or real antenna and the transfer efficiency of the tank circuit can readily be determined.

(*f*) *Dielectric Losses at High Frequency*

Preliminary experiments have been made on power factor of small condensers indicating that it may be possible to make phase-angle measurements on dielectrics of standard test size inserted between the plates of a two-plate condenser. It is not likely that these measurements will be as accurate as those obtained by other means already in use at this Laboratory, but they will be very rapid. This work has not progressed to the point where we can speak of it with certainty.

(*g*) *Percentage of Modulation*

This has already been mentioned earlier in this report. Ordinary forms of modulation produce the ellipse within the ellipse, the intervening area between the two ellipses being filled with diffused light. Since the inner ellipse represents a minimum power and the outer one maximum power during modulation, the percentage of modulation is readily computed.

(*h*) *Harmonic Power Analyses*

The distribution of harmonic power in an antenna or put out from a tube used as a frequency multiplier and operating into any kind of a load may be determined within certain limits. This work is still under way and at the present time it looks as though it would be difficult to make any reasonable determination of harmonic content smaller than three per cent. To make this determination the wattmeter is connected in the usual manner appropriate to the circuit in question and fundamental power is measured in the usual way. To get the harmonic power, the sensitivity of both voltage and current legs is increased until the

over-all sensitivity of the wattmeter is perhaps a hundredfold what it was for the measurement of fundamental power. Proper plug-in coils are put in corresponding to various harmonic frequencies. By the aid of a small driver of good wave form, the wattmeter is adjusted as regards phase set and calibrated for each of these harmonics. The power and phase angle can be determined for any of them within the limits of measurement. The procedure is also valuable in adjusting a transmitter for maximum fundamental power into a given antenna espe-

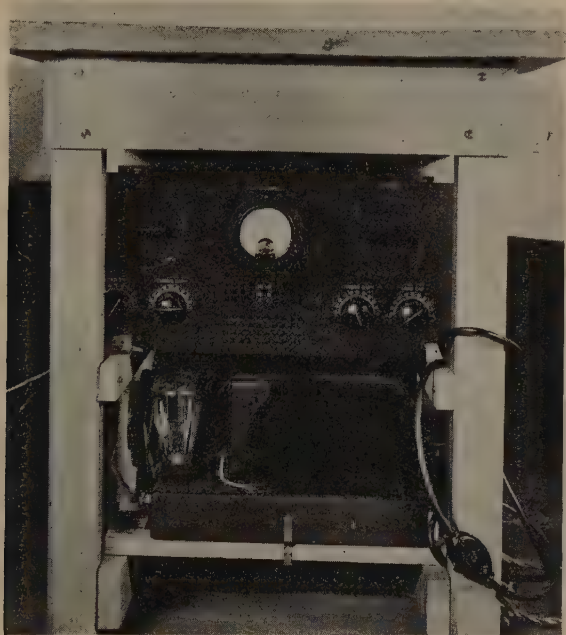

Fig. 7—Another view of the wattmeter proper showing arrangement of tuning controls on amplifiers.

cially if that antenna be of fixed dimension and not tunable. In this case the antenna ammeter does not give a true indication of maximum energy on the fundamental. Often the situation arises where an increase in coupling will cause a transmitter to draw more power and to put out more current in the antenna, whereas the wattmeter actually shows that the fundamental power has gone down. This is only another illustration of the fallacy of using nothing but an ammeter for this type of adjustment. A qualitative analysis of the number and relative strength of the harmonics may best be had by leaving the voltage leg tuned to the fundamental while the current leg is increased greatly in sensitivity

and then tuned through various harmonics as high perhaps as seven. The Lissajous figures produced on the screen readily tell us what har-

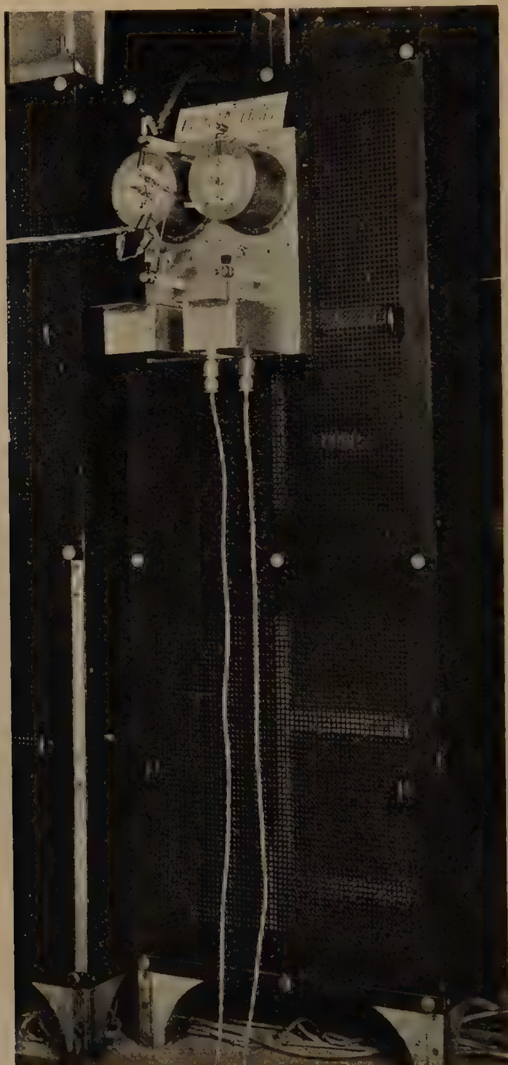

Fig. 8—Pickup unit. Current transformers at top, voltage pick-off condensers at bottom. Connection by transmission lines to wattmeter proper.

monics are predominant and the amplitude of the deflection on the current axes gives us a rough indication of the relative amplitude of these harmonics. If we take the trouble to calibrate the leg current on each

harmonic, we have of course the absolute amplitude of the harmonic currents. However, the harmonic which has the largest current may by no means show the largest power as it may be badly out of phase with the corresponding harmonic voltage. Ten years ago a good many transmitters were still in use which used frequency doubling or tripling tubes in the output stage. One of these was set up and its output into a long antenna was examined. In this case although this transmitter working into a pure resistance load showed no harmonics in excess of

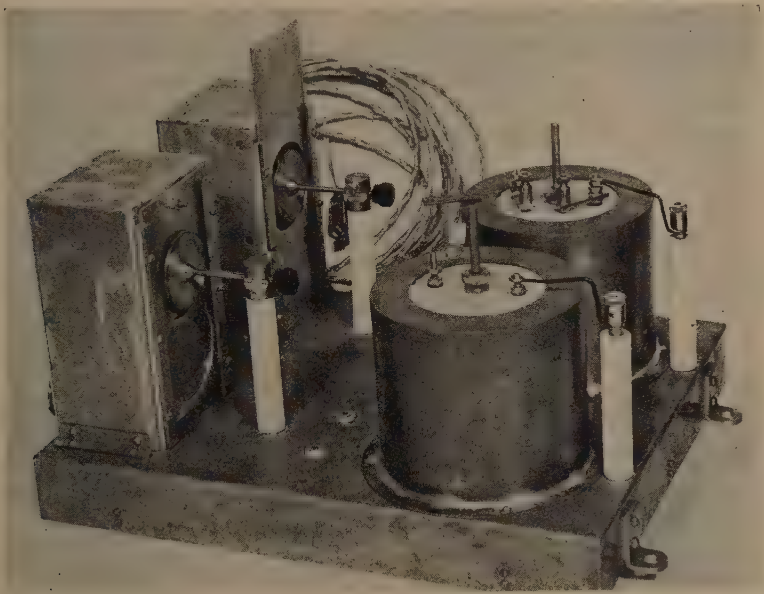

Fig. 9—Details of pickup unit.

three per cent, working into an actual antenna it showed not only a considerable third harmonic but a strong subharmonic corresponding to the frequency of excitation of the grid of the last tube in the transmitter (which was operating at the double frequency). Of course, with modern apparatus, subharmonics are not likely to be found. Other applications for the wattmeter are being investigated.

(*i*) *Determination of Q*

The best way of determining the Q of a coil with the wattmeter is to neutralize its inductance at the frequency in question with a series capacitance so that the power factor of the coil used as a load (see Fig. 2) is approximately unity. This gives a good figure easy to measure

and sufficient current can then be sent through the coil to get a good measurement. Knowing this current and calibrating the power from the measured area and knowing the value of the neutralized capacitance, we have all the necessary data. It must be admitted, however, that the method is neither as accurate nor as rapid as a Q meter especially when used with very high Q coils.

Conclusion

It is concluded that within the range up to thirty megacycles and possibly somewhat higher, the cathode-ray wattmeter is a very useful instrument provided not too high an accuracy is desired. It permits attacking certain problems which are either impossible of solution with existing instruments or involve very complicated circuit arrangements and laborious calculations. It permits solving other problems, perhaps not as well as by some other method but with far greater rapidity. The assembly as a whole constitutes not only a high-frequency wattmeter but a high-frequency ammeter and voltmeter and may readily be arranged so that the oscillograph itself can be used for any of the other purposes for which that instrument is adapted.

Acknowledgment

I want to express my thanks to Mr. E. L. Luke who has made many of the measurements referred to and is largely responsible for the evolution of the instrument from a rather complicated laboratory setup to a compact portable form. I wish also to thank Mr. Varela and Mr. Carlson for their assistance in building the driver associated with the instruments for convenience of measurement and for making many of the tests, calibrations, and measurements.

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THE PROPAGATION OF RADIO WAVES OVER THE SURFACE OF THE EARTH AND IN THE UPPER ATMOSPHERE*

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PART I

Ground-Wave Propagation from Short Antennas

Summary—*Simple formulas and graphs are given which represent the ground-wave field intensity at the surface of the earth as radiated from a short vertical antenna at the surface of the earth. The theory is compared to some experimental results reported by other investigators to determine its range of application. The diffraction formula given is theoretically valid only at the lower frequencies; however, it was shown that sky waves are important both day and night and over land and sea at those distances where diffraction would otherwise cause a marked decrease in the received field intensity. The attenuation formula given for the short distances where diffraction may be nelgected is theoretically valid for any frequency and set of ground constants; experimental data are given which show that the formula may be used even at the ultra-high frequencies.*

1. Introduction

In 1909 Sommerfeld[1] solved the general problem of the effect of the finite conductivity of the ground on the radiation from a grounded condenser antenna. Since that time many other investigators have obtained similar solutions of the problem in various ways. However, very few of these results have been left in a form convenient for engineering use. It is the purpose ofthis paper to reduce the complex equations of the Sommerfeld theory to the form of simple formulas and graphs which may readily be used by the engineer and to show their limitations by comparing them to the available experimental data.

It should be emphasized at this point that no formulas for field intensity can replace measurement data. All of the formulas presented here have been checked in certain frequency and distance ranges and represent the results of experiment fairly well in those ranges. When the theory is used to predict the field intensity in other frequency and distance ranges than those for which it has been checked, then it must be remembered that there is a possibility of error due to leaving out some factor which becomes important in these new situations.

* Decimal classification: R113.7. Original manuscript received by the Institute, July 31, 1936. This paper represents work which was begun at the National Bureau of Standards and completed at the Federal Communications Commission.

[1] *Ann. der. Phys.* vol. 28, p. 665, (1909).

The formula for the field intensity F (millivolts per meter) at a distance d (miles) from an antenna in which the current is I_0 (amperes) may be written

$$F = \frac{37.28\, k h_e I_0 A_1}{d} \tag{1}$$

where $k = 2\pi/\lambda$, A_1 is the attenuation factor, h_e is the effective height, and λ is the wave length. h_e and λ are to be measured in the same units and h_e and I_0 are referred to the same point on the antenna, which is the point at which the current is measured. The effective height of an antenna is determined by its physical dimensions, the current distribution, the ground constants, and the direction (in both the horizontal, and vertical planes) in which the transmission (or reception) takes place.

When the effect of the finite conductivity of the ground may be neglected, the effective height of a grounded vertical antenna of height h may be expressed by

$$h_e = \frac{\cos\psi}{I_0}\int_0^h I \cos(ka \sin\psi) da \tag{2}$$

where ψ is the angle above the horizon with which the waves leave the antenna and I is the current at any height [2] a. More general formulas for the effective height will be given in Part II which include the effect of the finite conductivity of the ground. At the same time formulas will be derived for the attenuation factor.

2. The Ground-Wave Radiation from Short Grounded Vertical Antennas over a Plane Earth

In case the antenna has a height much less than a wave length, the effective height for ground-wave radiation is equal to half the physical height with no top loading or equal to the physical height in case the antenna has sufficient top loading so that the current is uniform throughout the vertical portion of the antenna. In this latter case the following formula may be used for the attenuation factor for the ground-wave field intensity at the surface of the earth which is assumed to be flat:

[2] For a discussion of (2) as applied to broadcast antennas see G. H. Brown, Proc. I.R.E., vol. 24, pp. 48–91; January, (1936) (In Brown's notation $h_e = Kf(\theta)/k$.) For a theoretical and experimental proof that the effective height for receiving is equal to the effective height for transmitting see R. M. Wilmotte, *Phil. Mag.*, vol. 4, (1927) or Collected Researches of the National Physical Laboratory, vol. 22, pp. 21–32, (1930).

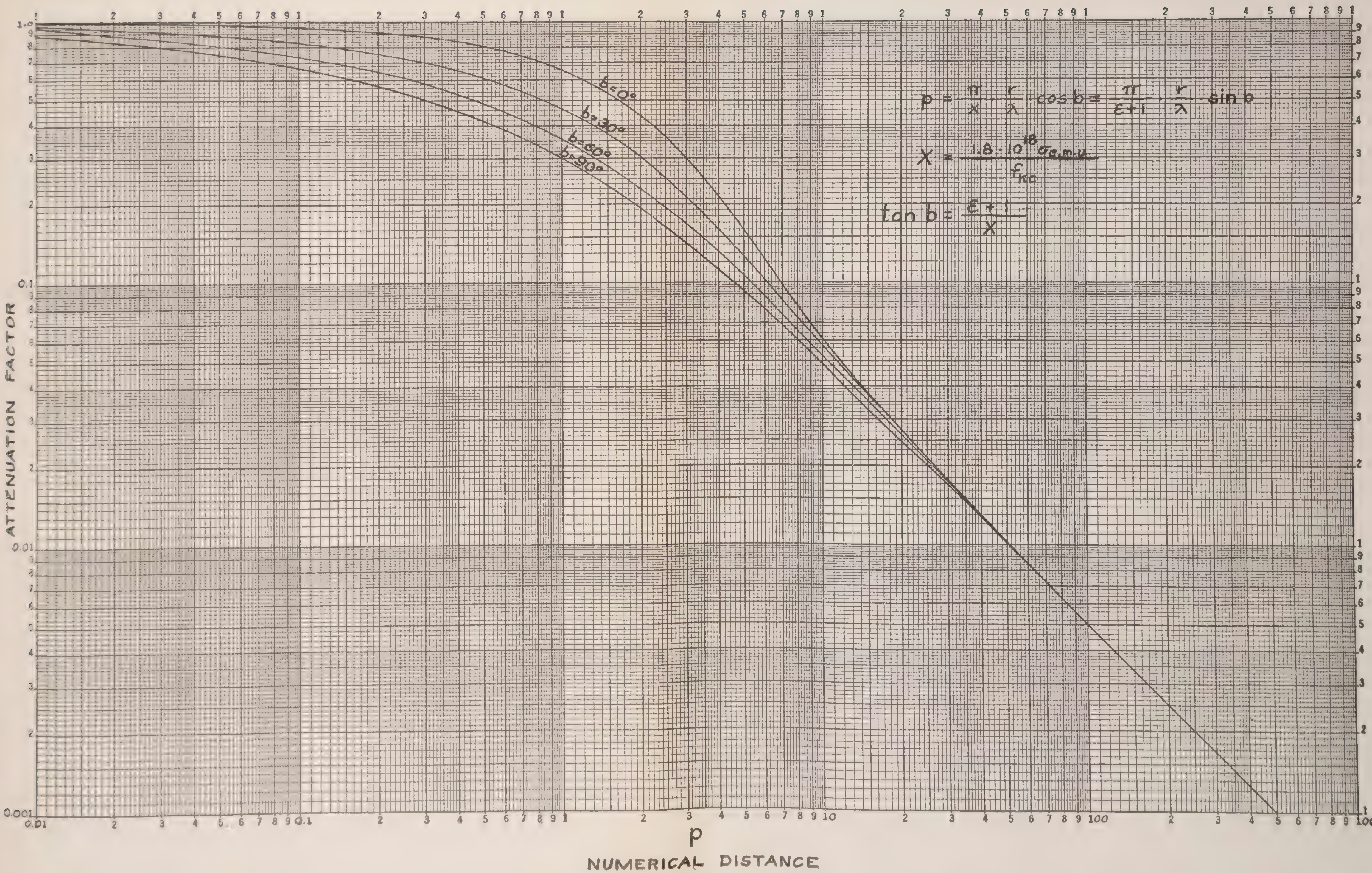

Fig. 1—Ground-wave attenuation factor as a function of the parameters p and b.

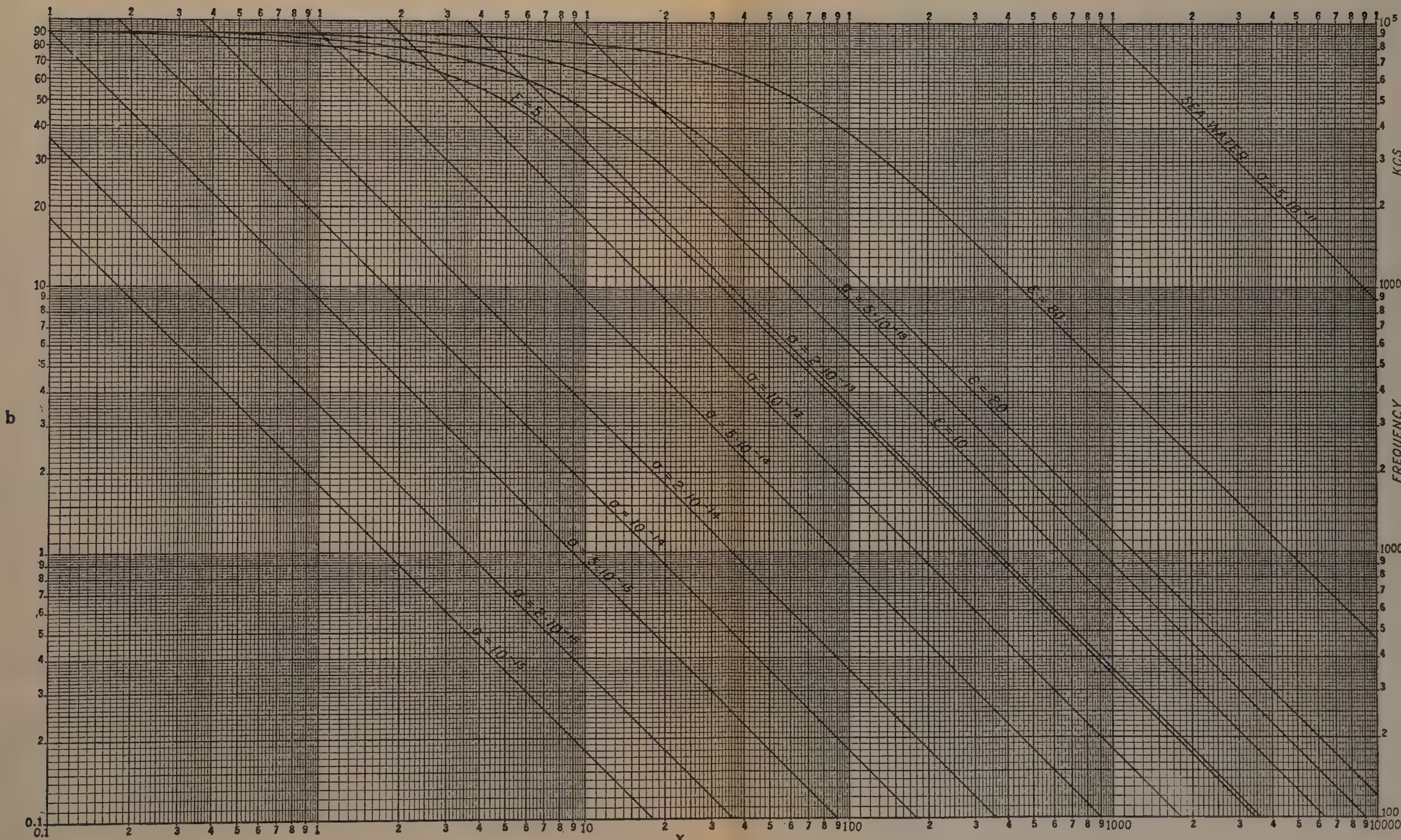

Fig. 2—The parameters x and b as a function of the frequency, conductivity, and dielectric constant.

$$A_1 = \left| 1 + i\sqrt{\pi p_1}\, e^{-p_1}\, erfc\,(-i\sqrt{p_1}) \right| \tag{3}$$

where,

$$p_1 = pe^{ib} \tag{4}$$

$$p = \frac{\pi}{x}\,\frac{r}{\lambda}\cos b = \frac{\pi}{\epsilon + 1}\,\frac{r}{\lambda}\sin b \tag{5}$$

$$\tan b = (\epsilon + 1)/x \tag{6}$$

$$x = 1.8 \cdot 10^{18} \sigma_{\text{e.m.u.}}/f_{kc} \tag{7}$$

$$erfc\,(-i\sqrt{p_1}) = \frac{2}{\sqrt{\pi}} \int_{-i\sqrt{p_1}}^{\infty} e^{-x^2} dx = i\frac{2}{\sqrt{\pi}} \int_{+i\infty}^{\sqrt{p_1}} e^{x^2} dx. \tag{8}$$

The derivation of (3) is given in the Appendix indicating the approximations made and the assumptions necessary to obtain a solution in terms of only two parameters, p and b. In (5) r/λ denotes the distance from the transmitting antenna measured in wave lengths. ϵ is the dielectric constant of the ground referred to air as unity, σ is the conductivity of the ground measured in electromagnetic units, and f is the frequency in kilocycles per second.

Equation (3) is given in Table I and graphically in Fig. 1 in terms of the two parameters p and b. The parameter p is called the "numerical distance." Fig. 2 shows the parameters x and b as a function of frequency, conductivity, and dielectric constant. x is determined by the frequency-conductivity intersections and having determined x, b is determined by the ϵ, x intersections.

Van der Pol[3] has given an empirical formula which is a fair approximation to (3) when $b < 5$ degrees:

$$A_{10} \cong \frac{2 + 0.3p}{2 + p + 0.6p^2} \tag{9}$$

where,

$$b < 5 \text{ degrees}$$

and the author[4] gave a correction factor to (9) (determined empirically from (3)) which extends its range of application to all values of b:

$$A_1 \cong A_{10} - \sin b\sqrt{p/2}\, e^{-5/8p}. \tag{10}$$

Two other empirical formulas which are useful in some applications are

$$A_{10} \cong e^{-0.43p + 0.01p^2} \tag{11}$$

[3] *Jahr. der Draht. Tel. und Tel.*, vol. 37, pp. 152–156, (1931).
[4] *Nature*, vol. 135, p. 954; June 8, (1935).

TABLE I

p	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	60°	70°	90°
0.02	0.9914	0.981	0.970	0.960	0.949	0.939	0.930	0.920	0.911	0.903	0.894	0.878	0.864	0.8377
0.04	0.9829													0.7786
0.05	0.9788	0.962	0.946	0.930	0.915	0.901	0.887	0.873	0.860	0.847	0.835	0.812	0.791	0.7577
0.06	0.9745													0.7363
0.1	0.9581	0.935	0.913	0.892	0.872	0.853	0.834	0.816	0.799	0.784	0.768	0.741	0.716	0.6740
0.24	0.9010													0.5447
0.25	0.8987	0.866	0.835	0.807	0.780	0.755	0.731	0.708	0.688	0.669	0.650	0.617	0.589	0.5382
0.26	0.8950													0.5316
0.50	0.8085	0.769	0.733	0.699	0.668	0.640	0.613	0.589	0.566	0.547	0.527	0.494	0.465	0.4200
0.74	0.731													0.3516
0.75	0.727													0.3495
0.76	0.725													0.3470
1	0.6567	0.616	0.579	0.546	0.516	0.489	0.465	0.443	0.423	0.406	0.389	0.361	0.338	0.3003
1.5	0.53[illegible]													0.2357
2	0.4398	0.408	0.381	0.357	0.336	0.317	0.301	0.286	0.272	0.261	0.250	0.232	0.217	0.1937
2.5	0.3635	0.338	0.316	0.297	0.280	0.265	0.251	0.240	0.229	0.220	0.211	0.196	0.184	0.1644
3	0.3027	0.282	0.264	0.249	0.236	0.219	0.213	0.204	0.195	0.188	0.180	0.168	0.158	0.1427
4	0.2154	0.203	0.192	0.182	0.174	0.166	0.159	0.153	0.148	0.143	0.139	0.131	0.124	0.1126
5	0.1593	0.152	0.146	0.140	0.135	0.131	0.127	0.123	0.119	0.116	0.113	0.108	0.103	0.09281
6	0.1228													
7	0.0984													
8	0.0816	0.0804	0.0791	0.0778	0.0765	0.0749	0.0738	0.0725	0.0712	0.0699	0.0686	0.0662	0.0640	0.06036
9	0.0696													0.05401
10	0.0606	0.0604	0.0602	0.0599	0.0595	0.0588	0.0577	0.0567	0.0556	0.0545	0.0535	0.0515	0.0500	0.04884
11	0.0539													
12	0.0485													
13	0.0441	0.0441	0.0441	0.0440	0.0439	0.0438	0.0436	0.0433	0.0430	0.0425	0.0420	0.0407	0.0392	0.03788
15	0.0374													
18	0.0305	0.0305	0.0305	0.0305	0.0305	0.0305	0.0305	0.0304	0.0304	0.0302	0.0301	0.0295	0.0288	0.02753
20	0.0272													0.02484
30	0.0176													
40	0.0130													
50	0.01032	0.0103	0.0103	0.0103	0.0103	0.0103	0.0103	0.0103	0.0103	0.0103	0.0103	0.0103	0.0103	0.01030

where,

$$b < 5 \text{ degrees and } p \leqq 4.5$$

$$A_{10} \cong \frac{1}{2p - 3.7} \tag{12}$$

where,

$$b < 5 \text{ degrees and } p \geqq 4.5.$$

The exponential character of the attenuation at short "numerical distances" as exhibited by (11) is made use of in determining the un-

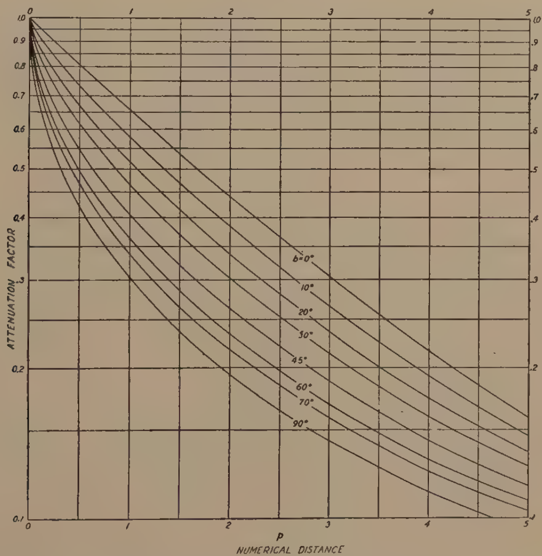

Fig. 3—Ground-wave attenuation factor as a function of the parameters p and b.

absorbed field intensity at a unit distance by plotting field intensity times distance on a logarithmic scale versus distance on a linear scale. The unabsorbed field intensity at unit distance is the zero distance ordinate determined by extrapolating the data linearly to zero distance. However, unless b is very nearly equal to zero this exponential relation will not hold as is illustrated in Fig. 3 which gives (3) on semi-logarithmic graph paper.

The first term in the asymptotic expansion of

$$i\sqrt{\pi p_1}e^{-p_1}\, erfc\,(-\, i\sqrt{p_1}) = -1 - \frac{1}{2p_1} \tag{13}$$

and when this is substituted in (3) we obtain

$$A_1 = \left| -\frac{1}{2p_1} \right| = \frac{1}{2p} \tag{14}$$

where,

$$p > 20.$$

Equation (14) is valid for any value of b, being valid for shorter numerical distances (see Fig. 1) when b is large. At the intermediate frequencies, i.e., from 1500 to 5000 kilocycles, C. N. Anderson[5] has published ground-wave attenuation measurements for transmission over land which

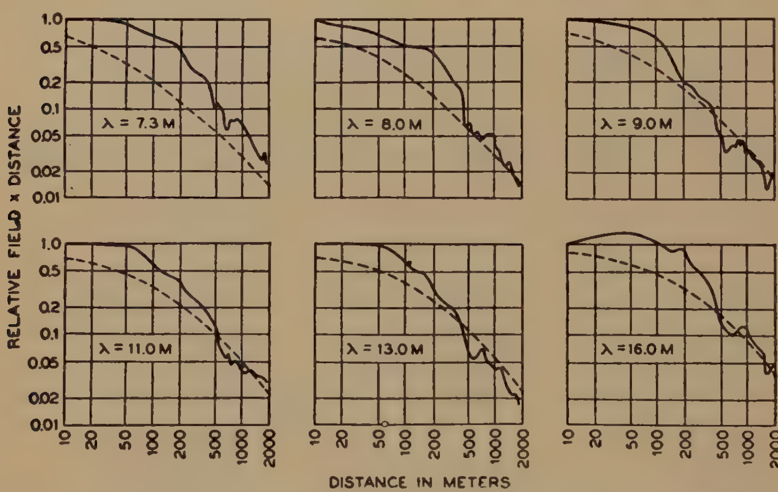

Fig. 4—Experimental and theoretical ground-wave attenuation data at high frequencies. (The theoretical curves are dotted.)

clearly indicated the importance of (14). At still higher frequencies C.B. Feldman[6] has reported some very carefully conducted experiments which determined the attenuation over level land. These data are given in Fig. 15 of his paper and are reproduced as Fig. 4 of the present paper together with the theoretical graphs (dotted) for a dielectric constant of 20 and conductivity 2×10^{-13} electromagnetic units. These are the values of the ground constants obtained by Feldman using an independent method of determination. The agreement is qualitatively very good and would be still better if the measured values were shifted downward which is evidently the correct procedure in this case since the measured attenuation factor at a distance of ten meters was arbitrarily set equal to one for all frequencies. The finite heights of the

[5] Proc. I.R.E., vol. 21, p. 1447; October, (1933).
[6] Proc. I.R.E., vol. 21, pp. 765–801; June, (1933).

transmitting and receiving antennas may cause the remaining discrepancy between theory and experiment at near distances. This will be discussed in Part II.

3. The Effect of the Curvature of the Earth on Ground-Wave Attenuation

The Watson diffraction formula[7] for the attenuation around a perfectly conducting spherical earth may be expressed by

$$A_2 = A_M z^{1/2} \sum_{1}^{\infty} a_n e^{-1/2(b_n z - 1)} \tag{15}$$

where,

$$A_M = 0 \cdot 7944$$

$$z = 1.808 \cdot 10^{-3} f_{kc}{}^{1/3} \cdot d \tag{16a}$$

and the constants a_n and b_n are given in Table II. A_2 is given as a function of z in Table III.

TABLE II
(These Constants are Taken from C. R. Burrows[8])

n	a_n	b_n
1	1	1
2	0.3135	3.189
3	0.2110	4.734
4	0.1653	6.037
5	0.1382	7.234
6	0.1199	8.324

Burrows[8] has shown that to include the effect of refraction in the lower atmosphere it is only necessary to use a radius of the earth equal to 4/3 the actual radius which has the effect of reducing z by the factor $(3/4)^{2/3}$ so that (16a) becomes

$$z = 1.492 \cdot 10^{-3} f_{kc}{}^{1/3} \cdot d. \tag{16b}$$

TABLE III
The Diffraction Attenuation Factor

z	A_2	z	A_2
0	1	10	$2.79 \cdot 10^{-2}$
0.75	0.97	11	$1.77 \cdot 10^{-2}$
1	0.924	12	$1.12 \cdot 10^{-2}$
1.5	0.816	13	$7.10 \cdot 10^{-3}$
2	0.710	14	$4.47 \cdot 10^{-3}$
2.5	0.606	15	$2.80 \cdot 10^{-3}$
3	0.512	16	$1.76 \cdot 10^{-3}$
3.5	0.429	17	$1.10 \cdot 10^{-3}$
4	0.356	18	$6.86 \cdot 10^{-4}$
5	0.241	19	$4.27 \cdot 10^{-4}$
6	0.160	20	$2.66 \cdot 10^{-4}$
7	0.105	21	$1.65 \cdot 10^{-4}$
8	$6.78 \cdot 10^{-2}$	22	$1.03 \cdot 10^{-4}$
9	$4.36 \cdot 10^{-2}$		

[7] G. N. Watson, *Proc. Roy. Soc.* (London), A, vol. 95, pp. 83–99; (1918).
[8] Proc. I.R.E., vol. 23, pp. 470–480; May, (1935).

T. L. Eckersley[9] has shown that the effect of the finite conductivity of the earth may be included by increasing z by the factor $K/23.9$ so that (16b) becomes

$$z = 1.492 \cdot 10^{-3} f_{kc}^{1/3} \cdot d \frac{K}{23.9} \cdot \tag{16}$$

The factor K is the function of the ground constants given in Fig. 2 of Eckersley's paper. It is given in Table IV as a function of the parameter β:

$$\beta = 1.72 \cdot 10^{20} \sigma_{\mathrm{e.m.u}} / f_{kc}^{5/3}. \tag{17}$$

Eckersley's method does not determine the value of the constant factor A_M in (15) which is equal to 0.7944 for a perfectly conducting earth.

TABLE IV

Parameters for the Watson-Eckersley Diffraction Formula ($b=0$; to Include the Effect of Refraction in the Lower Atmosphere, A Radius of the Earth 4/3 the Actual Radius was Used)

β	K	p_m	p_x	A_M
10^{-4}	53.0	$4.86 \cdot 10^{6}$	$1.36 \cdot 10^{7}$	$5.27 \cdot 10^{-8}$
$2 \cdot 10^{-4}$	53.0	$2.43 \cdot 10^{6}$	$6.80 \cdot 10^{6}$	$1.05 \cdot 10^{-7}$
$5 \cdot 10^{-4}$	52.9	$9.74 \cdot 10^{5}$	$2.73 \cdot 10^{6}$	$2.63 \cdot 10^{-7}$
10^{-3}	52.8	$4.88 \cdot 10^{5}$	$1.37 \cdot 10^{6}$	$5.25 \cdot 10^{-7}$
$2 \cdot 10^{-3}$	52.7	$2.44 \cdot 10^{5}$	$6.83 \cdot 10^{5}$	$1.05 \cdot 10^{-6}$
$5 \cdot 10^{-3}$	52.6	$9.80 \cdot 10^{4}$	$2.74 \cdot 10^{5}$	$2.61 \cdot 10^{-6}$
10^{-2}	52.5	$4.91 \cdot 10^{4}$	$1.37 \cdot 10^{5}$	$5.22 \cdot 10^{-6}$
$2 \cdot 10^{-2}$	52.4	$2.46 \cdot 10^{4}$	$6.89 \cdot 10^{4}$	$1.04 \cdot 10^{-5}$
$5 \cdot 10^{-2}$	52.3	$9.85 \cdot 10^{3}$	$2.76 \cdot 10^{4}$	$2.60 \cdot 10^{-5}$
10^{-1}	52.2	$4.93 \cdot 10^{3}$	$1.38 \cdot 10^{4}$	$5.20 \cdot 10^{-5}$
$2 \cdot 10^{-1}$	52.1	$2.47 \cdot 10^{3}$	$6.92 \cdot 10^{3}$	$1.03 \cdot 10^{-4}$
$5 \cdot 10^{-1}$	52.0	$9.90 \cdot 10^{2}$	$2.77 \cdot 10^{3}$	$2.59 \cdot 10^{-4}$
1	51.9	$4.96 \cdot 10^{2}$	$1.39 \cdot 10^{3}$	$5.16 \cdot 10^{-4}$
2	51.8	$2.49 \cdot 10^{2}$	$6.97 \cdot 10^{2}$	$1.03 \cdot 10^{-3}$
5	51.6	99.8	$2.80 \cdot 10^{2}$	$2.56 \cdot 10^{-3}$
10	51.3	50.2	$1.40 \cdot 10^{2}$	$5.10 \cdot 10^{-3}$
20	50.8	25.3	71.0	$1.01 \cdot 10^{-2}$
50	49.7	10.4	29.0	$2.58 \cdot 10^{-2}$
10^{2}	48.5	5.31	15.7	$5.47 \cdot 10^{-2}$
$2 \cdot 10^{2}$	46.8	2.75	9.70	0.119
$5 \cdot 10^{2}$	43.2	1.19	4.20	0.377
10^{3}	39.3	$6.56 \cdot 10^{-1}$	$9.50 \cdot 10^{-1}$	0.642
$2 \cdot 10^{3}$	34.8	$3.70 \cdot 10^{-1}$	$2.85 \cdot 10^{-1}$	0.724
$5 \cdot 10^{3}$	29.5	$1.74 \cdot 10^{-1}$	$9.00 \cdot 10^{-2}$	0.766
10^{4}	27.5	$9.37 \cdot 10^{-2}$	$4.15 \cdot 10^{-2}$	0.779
$2 \cdot 10^{4}$	26.3	$4.90 \cdot 10^{-2}$	$2.04 \cdot 10^{-2}$	0.787
$5 \cdot 10^{4}$	25.4	$2.03 \cdot 10^{-2}$	$8.12 \cdot 10^{-3}$	0.794
10^{5}	25.0	$1.03 \cdot 10^{-2}$	$4.12 \cdot 10^{-3}$	0.794
$2 \cdot 10^{5}$	24.7	$5.22 \cdot 10^{-3}$	$2.09 \cdot 10^{-3}$	0.794
$5 \cdot 10^{5}$	24.5	$2.10 \cdot 10^{-3}$	$8.40 \cdot 10^{-4}$	0.794
10^{6}	24.4	$1.05 \cdot 10^{-3}$	$4.22 \cdot 10^{-4}$	0.794
$2 \cdot 10^{6}$	24.3	$5.30 \cdot 10^{-4}$	$2.12 \cdot 10^{-4}$	0.794
$5 \cdot 10^{6}$	24.2	$2.13 \cdot 10^{-4}$	$8.52 \cdot 10^{-5}$	0.794
10^{7}	24.1	$1.07 \cdot 10^{-4}$	$4.28 \cdot 10^{-5}$	0.794
$2 \cdot 10^{7}$	24.0	$5.36 \cdot 10^{-5}$	$2.14 \cdot 10^{-5}$	0.794
$5 \cdot 10^{7}$	23.9	$2.15 \cdot 10^{-5}$	$8.62 \cdot 10^{-6}$	0.794
10^{8}	23.9	$1.08 \cdot 10^{-5}$	$4.32 \cdot 10^{-6}$	0.794

However, the more complete solution of the problem as given by B. Wwedensky[10] indicates that the diffraction formula gives values lying below the Sommerfeld plane earth solution (equation (3)) at all distances but approaching this latter solution asymptotically at the

[9] Proc. I.R.E., vol. 20, pp. 1555–1579; October, (1932).
[10] *Tech. Physics* (U.S.S.R.), vol. 2, no. 6, pp. 624-639, (1935). (In English.)

shorter distances. Wwedensky also obtained values of K that are approximately in agreement with those given by Eckersley. Thus Eckersley's[11] recent suggestion that his curves should be shifted vertically until they are tangent or nearly tangent to the Sommerfeld curve has been placed on a sound theoretical basis and this process may be adopted for an approximate solution of the diffraction problem.

Let p_m denote the value of the numerical distance corresponding to $z=1$. Equations (16), (17), and (5) may be solved for p_m when

$$p_m = 2.577 \cdot 10^4 \cos b / K\beta. \tag{18}$$

p_m is given as a function of β for $b=0$ in Table IV. Let p_x denote the

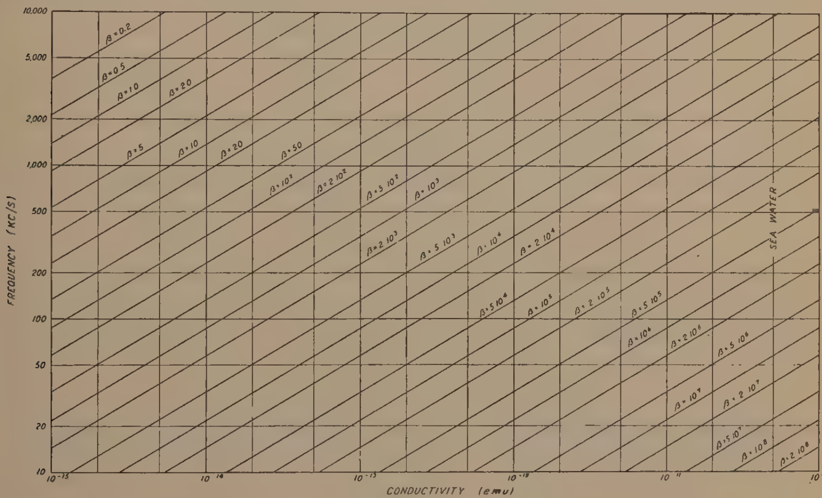

Fig. 5—The parameter β as a function of the frequency and conductivity.

value of the numerical distance at which A_1 is equal to A_2 both in magnitude and slope. This value may be determined most easily by plotting A_1 as a function of p on log-log graph paper and then laying the plot of A_2 as a function of z on top. For a given value of β the numerical distance for $z=1$, i.e., p_m, is determined. This fixes the relation between the abscissas on the two charts. Now the A_2 chart is shifted vertically until the two curves are just tangent. p_x is the value of p at the point of tangency and the new value of A_M may be determined also by multiplying the ratio of the ordinates on the two charts by 0.7944. The resulting values of A_M and p_x are given in Table IV as a function of β and β is shown as a function of frequency and conductivity in Fig. 5.

[11] Document A.G., (1934), U.R.S.I., Fifth Assembly.

Fig. 6 gives equation (3) for $b=0$ as a function of p together with a set of diffraction curves for various values of β. It is evident from this figure that when $\beta > 2000$ diffraction around rather than absorption in the earth is the controlling factor. Since $\beta > 2000$ over sea water for frequencies less than 10,000 kilocycles it is of interest to compare the empirical Austin-Cohen formula which applies to sea water transmis-

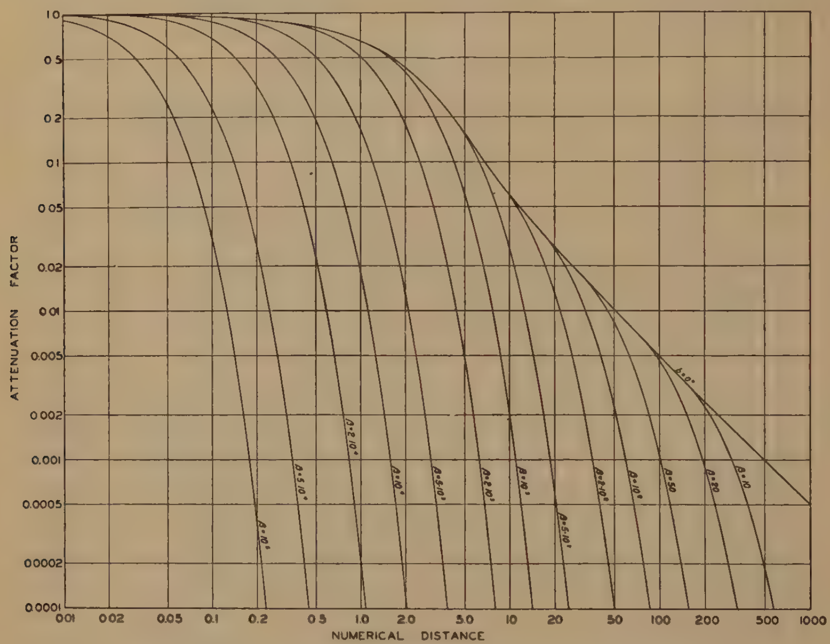

Fig. 6—Ground-wave attenuation factor as a function of the parameters p and β.

sion with the diffraction formula for this case. The Austin-Cohen formula may be written

$$A_e = e^{-\gamma d}. \tag{19}$$

The parameter γ has been determined[12] experimentally to be equal to

$$\gamma_e \doteq 1.39 f_{kc}^{1/2} \cdot 10^{-4}. \tag{20}$$

Fig. 7 shows the attenuation factor for sea water ($\sigma = 5 \cdot 10^{-11}$ e.m.u.) and the frequencies 10, 100, 1000, and 10,000 kilocycles as given by (15) with z defined by (16) and by the Austin-Cohen formula. It is evident that the Watson-Eckersley-Burrows diffraction formula is in

[12] C. N. Anderson, Proc. I.R.E., vol. 19, pp. 1150–1165; July, (1931).

fair agreement with the empirical Austin-Cohen formula, especially at the nearer distances. The tendency of the Austin-Cohen formula to give larger values of field intensity at the low frequencies and great distances is probably due to the fact that sky waves are important in the daytime at these distances. This is illustrated by the data reported by Anderson[12] for the case of sea water propagation. In the case of overland transmission in the broadcast band, Norton, Kirby, and Lester[13] reported that sky waves during the daytime began to be of importance at the numerical distance p_x at which diffraction should begin to influence the ground-wave intensity. Finally, in the ultra-

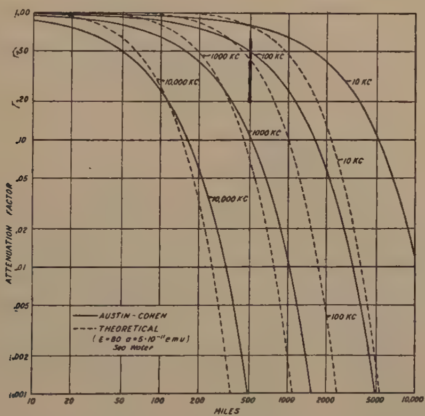

Fig. 7—A comparison of the Austin-Cohen and the theoretical ground-wave attenuation formulas for sea water.

high-frequency range, Burrows, Decino, and Hunt[14] have reported fading at the distance at which diffraction should be noticeable. Thus it may be said in general that the diffraction problem is of more theoretical than practical importance although there may be times or situations when the above analysis may be useful. In any case the equations given represent the minimum fields to be expected and the values of p_x represent the greatest numerical distances for which the Sommerfeld theory for a plane earth may be expected to apply.

The T. L. Eckersley correction for the effect of the finite conductivity of the earth was derived on the assumption that $b=0$. At the ultra-high frequencies $b=90$ degrees and this assumption is not justified. In this case Burrows, Decino, and Hunt[14] have given another diffraction formula which is represented in Fig. 10 of their paper.

[13] Proc. I.R.E., vol. 23, pp. 1183–1200; October, (1935).
[14] Proc. I.R.E., vol. 23, pp. 1507–1535; December, (1935).

4. Graphs Illustrating Ground-Wave Propagation from Short Antennas

Equation (1) may be expressed in terms of radiated power P_r (expressed in kilowatts) by substituting $\sqrt{1000\ P_r/R_r}$ for I_0 (where R_r is the radiation resistance in ohms):

$$F = \frac{37.28 k h_e 10\sqrt{10}\sqrt{P_r}A_1}{d\sqrt{R_r}}. \tag{21}$$

For a short antenna ($h \ll \lambda$) without top loading $h_e = h/2$ and[15] $R_r = 10\ (kh)^2$ and with a large amount of top loading $h_e = h$ and $R_r = 40\ (kh)^2$. In either case when these values are substituted into (21) we obtain

$$F = \frac{186.4\sqrt{P_r}A_1}{d} \tag{22}$$

where,

$$h << \lambda$$

and A_1 is given by (3) which is to be supplemented by (15).

Using (22), (3), and (15), graphs have been prepared showing the variation of ground-wave field intensity with distance for one kilowatt of radiated power, for the frequencies 150, 300, 550, 1000, 1500, 2000, 3000, and 5000 kilocycles. Fig. 8 is for a conductivity $\sigma = 5 \cdot 10^{-11}{}_{\mathrm{e.m.u.}}$ corresponding to sea water, Fig. 9 is for land of average conductivity $\sigma = 10^{-13}{}_{\mathrm{e.m.u.}}$ while Fig. 10 corresponds to poorly conducting land with $\sigma = 10^{-14}{}_{\mathrm{e.m.u.}}$. The ground wave is here considered to be that portion of the wave received on the earth's surface which has not been propagated by conducting portions of the upper atmosphere. At the greater distances, sky waves, much stronger than the ground waves here predicted, are observed both day and night.

("The Effects of the Transmitting Antenna Height and the Height of the Receiving Point on the Effective Height and the Attenuation Factor," together with formulas for ultra-high-frequency propagation, will be considered in Part II of this paper which, it is expected, will be published in a subsequent issue of the Proceedings.)

Note

Note added August 25, 1936: Some recent experimental results obtained by C. R. Burrows and described in a letter to *Nature*, vol 138, August 15, (1936), substantiate the theoretical ground-wave formulas and graphs given for the ultra-high frequencies (see equations (41c) and (44), and the ninety-degree curves). These measurements

[15] Stuart Ballantine, Proc. I.R.E., vol. 12, pp. 823–839; December, (1924).

were made on a frequency of 150,000 kilocycles over fresh water so that $\epsilon \gg x$. In his last sentence Mr. Burrows states: "It seems evident

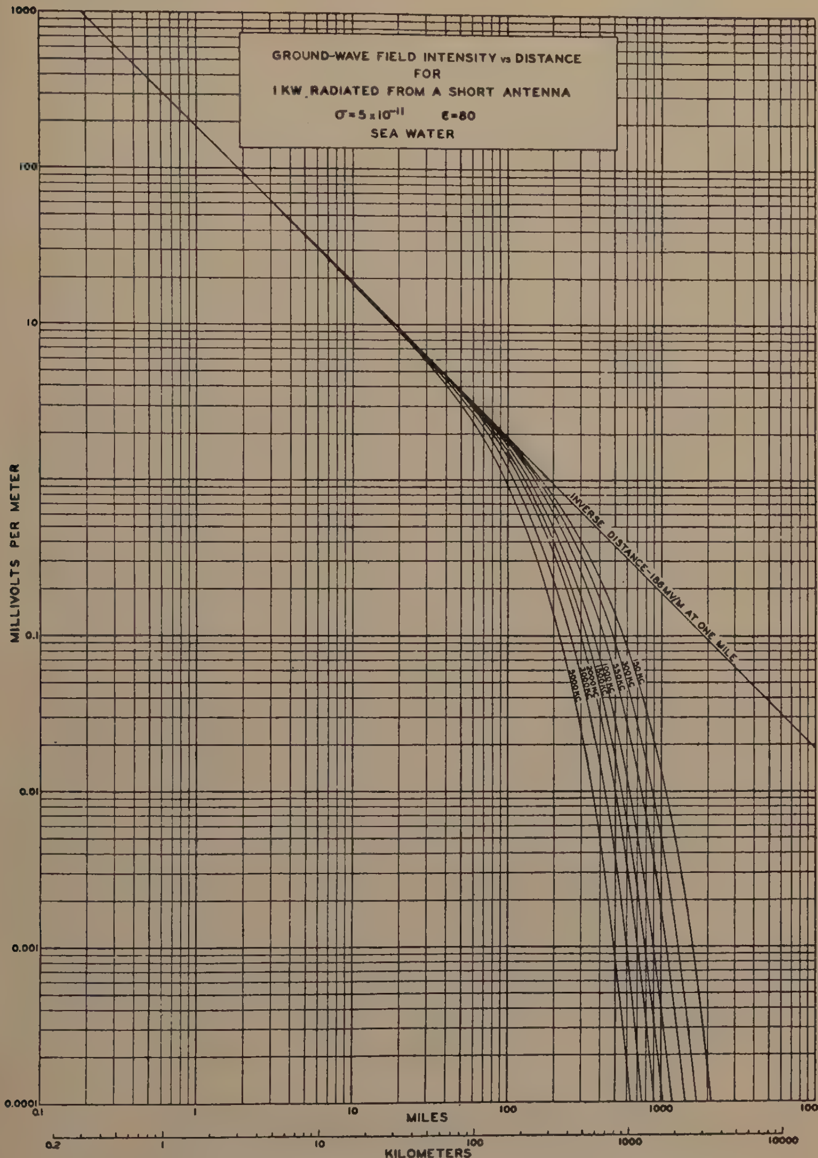

Fig. 8—Ground-wave field intensity vs. distance for sea water.

that a revision of the Sommerfeld-Rolf curves is required for propagation over all types of ground for which the dielectric constant cannot

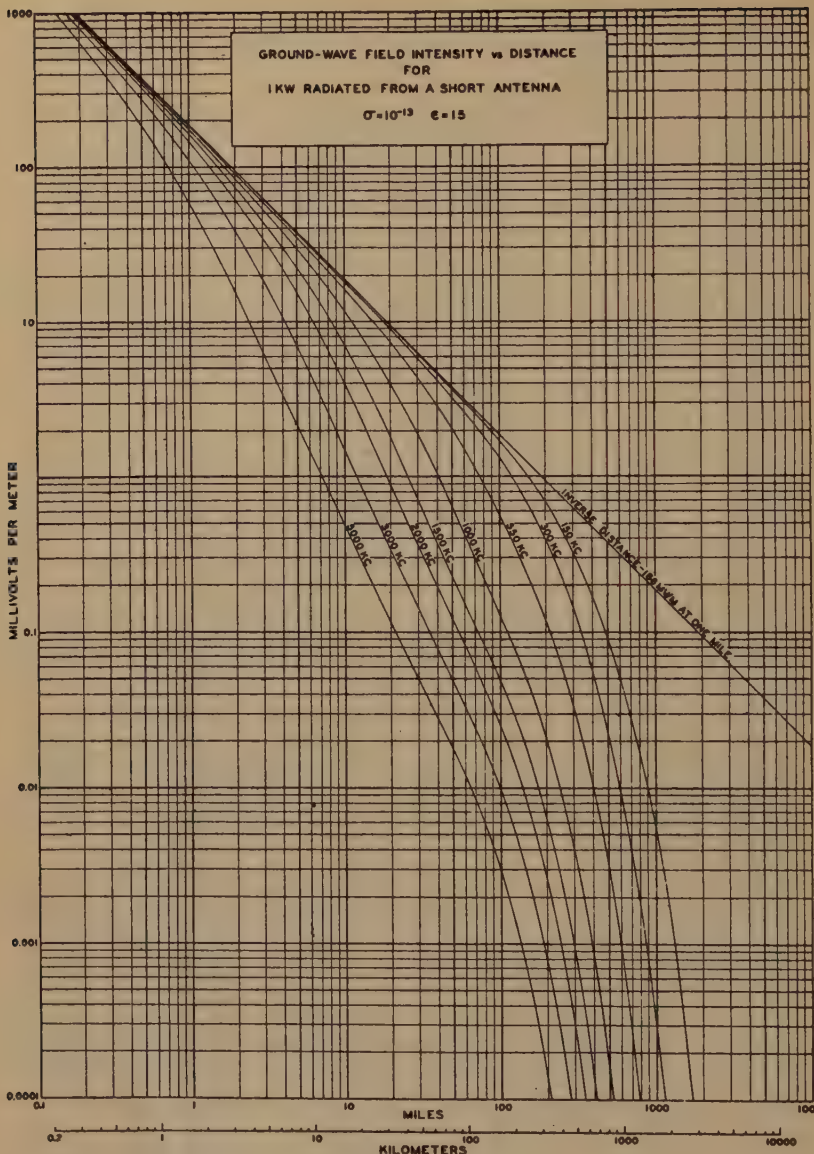

Fig. 9—Ground-wave field intensity vs. distance for land of average conductivity.

be neglected." Revised data of this kind are given in Fig. 1 and Table I of this paper and should be adequate in all cases such that

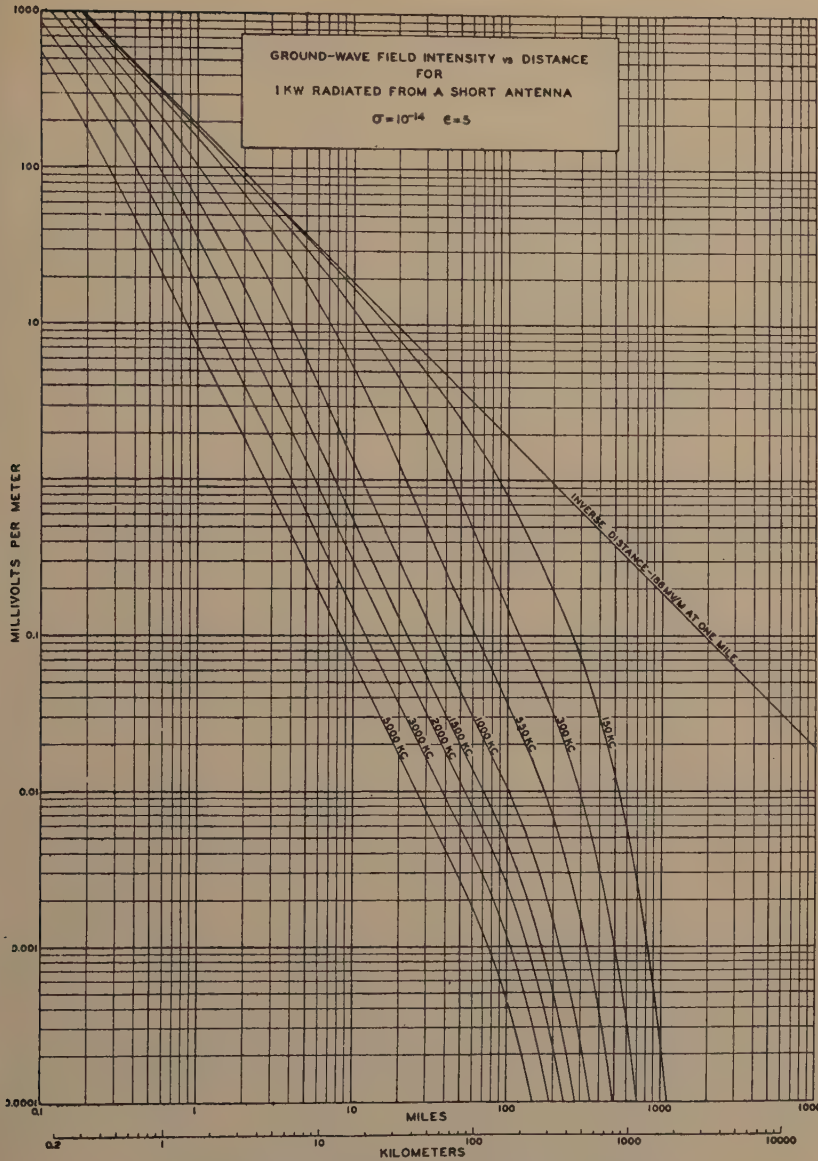

Fig. 10—Ground-wave field intensity vs. distance for land of low conductivity.

the heights of the transmitting and receiving antennas are a small fraction of a wave length above the surface of the earth.

Appendix

The Attenuation Factor for the Ground-Wave Radiation from Short Antennas

From the paper by B. van der Pol and K. F. Niessen,[16] we obtain the following exact expression for the ground-wave potential function of a vertical infinitesimal doublet:

$$\Pi(r,0) = \frac{e^{ikr}}{r}\,\frac{1}{1-y^4}\left[1 - y^2e^{p_2-p_1} - y\theta e^{-p_1}\int_{\sqrt{p_2}}^{\sqrt{p_1}} \frac{e^{w^2}dw}{(w^2+\theta)^{1/2}}\right] \tag{23}$$

$$y = (\epsilon + ix)^{-1/2} \tag{24}$$

$$p_1 = 2\pi i\left[1-(1+y^2)^{-1/2}\right]\frac{r}{\lambda} \equiv pe^{ib} \tag{25}$$

$$p_2 = 2\pi i\left[y^{-1}-(1+y^2)^{-1/2}\right]\frac{r}{\lambda} \tag{26}$$

$$\theta = 4\pi i(1+y^2)^{-1/2}\frac{r}{\lambda}. \tag{27}$$

The vertical electric field intensity may be determined most easily from (23) above by the use of (11) in W. H. Wise's paper:[17]

$$F = F_1'\left[\Pi/(1+y^2) + \frac{1}{1-y^2}\left(\frac{i}{kr} - \frac{1}{(kr)^2}\right)\frac{e^{ikr}}{r}\right] \tag{28}$$

and from the above equation we have for the attenuation factor:

$$A = (1+y^2)^{-1}(1-y^4)^{-1}\left[1-y^2e^{p_2-p_1}-y\theta e^{-p_1}\int_{\sqrt{p_2}}^{\sqrt{p_1}}\frac{e^{w^2}dw}{(w^2+\theta)^{1/2}}\right]$$
$$+(1-y^2)^{-1}\left(\frac{i}{kr}-\frac{1}{(kr)^2}\right). \tag{29}$$

Since the quantity in the square brackets of (29) is the only part which is difficult to evaluate, it will be considered alone and will be denoted by B. There are two cases to be considered in the evaluation of B.

Case I $|p_2| < |\theta|$ and $|p_1| < |\theta|$.

In this case the integrand in the integral in B may be expanded about the point $w=0$ and integrated by parts term by term giving

[16] *Ann. der Phys.*, 5 Folge, Band 6, Heft 3, pp. 273–294, (1930), equations (21) and (23a).

[17] Proc. I.R.E., vol. 19, pp. 1684–1689; September, (1931).

$$
\begin{aligned}
B = 1 - y^2 e^{p_2 - p_1} - y\theta^{1/2} e^{-p_1} \int_{\sqrt{p_2}}^{\sqrt{p_1}} e^{w^2} dw \Bigg[1 + \frac{1}{4\theta} + \cdots \\
+ \left\{ \frac{3\cdot 5\cdot 7 \cdots (2n-1)}{2^n} \right\}^2 \frac{1}{n!\theta^n} \Bigg] \\
+ \frac{y\theta^{1/2}}{2\sqrt{p_1}} \left[1 - \left(1 + \frac{p_1}{\theta} \right)^{-1/2} \right] \\
+ \frac{y\theta^{1/2}}{4\theta\sqrt{p_1}} \left[1 + \frac{p_1}{2\theta} - \frac{1 + 2p_1/\theta}{(1 + p_1/\theta)^{3/2}} \right] + \cdots \\
- \frac{y\theta^{1/2}}{2\sqrt{p_2}} \left[1 - \left(1 + \frac{p_2}{\theta} \right)^{-1/2} \right] e^{p_2 - p_1} \\
- \frac{y\theta^{1/2}}{4\theta\sqrt{p_2}} \left[1 + \frac{p_2}{2\theta} - \frac{1 + 2p_2/\theta}{(1 + p_2/\theta)^{3/2}} \right] e^{p_2 - p_1} - \cdots .
\end{aligned}
\tag{30}
$$

In the numerical calculation from (30), since p_2 always has a large negative real part and is always greater than 10 for $r > \lambda$, most of the terms will be negligibly small and we may replace $\sqrt{p_2}$ by $i\infty$ and obtain finally

$$
\begin{aligned}
B = 1 - \left(\frac{y^2\theta}{4p_1} \right)^{1/2} 2\sqrt{p_1} e^{-p_1} \int_{i\infty}^{\sqrt{p_1}} e^{w^2} dw \\
+ \left(\frac{y^2\theta}{4p_1} \right)^{1/2} \left[1 - \left(1 + \frac{p_1}{\theta} \right)^{-1/2} \right]
\end{aligned}
\tag{31}
$$

where,

$$r > \lambda.$$

Case II $|p_2| > |\theta|$ and $|p_1| < |\theta|$.

In this case the integral in B may be separated into two parts thus,

$$
\begin{aligned}
B = 1 - y^2 e^{p_2 - p_1} - y\theta e^{-p_1} \int_0^{\sqrt{p_1}} \frac{e^{w^2} dw}{(w^2 + \theta)^{1/2}} \\
+ y\theta e^{-p_1} \int_0^{\sqrt{p_2}} \frac{e^{w^2} dw}{(w^2 + \theta)^{1/2}} .
\end{aligned}
\tag{32}
$$

The first integral may be expanded as before and after the same changes are made as in Case I we obtain

$$
\begin{aligned}
B = 1 + y\theta e^{-p_1} \int_0^{i\infty} \frac{e^{w^2} dw}{(w^2 + \theta)^{1/2}} - \left(\frac{y^2\theta}{4p_1} \right)^{1/2} 2\sqrt{p_1} e^{-p_1} \int_0^{\sqrt{p_1}} e^{w^2} dw \\
+ \left(\frac{y^2\theta}{4p_1} \right)^{1/2} \left[1 - \left(1 + \frac{p_1}{\theta} \right)^{-1/2} \right]
\end{aligned}
\tag{33}
$$

where,

$$r > \lambda.$$

Now since $\theta >> w^2$ for small values of w and (along the imaginary axis) $e^{w^2} << 1$ for large values of w, the first integral is evidently approximately equal to

$$\theta^{-1/2} \int_0^{i\infty} e^{w^2} dw. \tag{34}$$

Substituting the above in (33) we have (31) as before. All physical applications are contained in these two cases.

It has been found in practice that ϵ is always greater than five so that $|y^4| < 0.04$ for any frequency or conductivity and we have

$$\left(\frac{y^2\theta}{4p_1}\right)^{1/2} = 1 + \frac{y^2}{8} - \frac{5}{128} y^4 + \cdots. \tag{35}$$

$$\left(\frac{y^2\theta}{4p_1}\right)^{1/2} [1 - (1 + p_1/\theta)^{-1/2}] = \frac{y^2}{8} + \frac{y^4}{64} + \cdots. \tag{36}$$

Now dropping terms of the order y^4 or greater, (31) becomes

$$B = \left(1 + \frac{y^2}{8}\right)\left[1 - 2\sqrt{p_1} e^{-p_1} \int_{i\infty}^{\sqrt{p_1}} e^{w^2} dw\right] \tag{37}$$

where,

$$r > \lambda, \ |y^4| \ll 1.$$

Since the second term in (29) is negligible when $r > \lambda$, (28) for the field intensity may be written

$$F = F_1'(1 + y^2)^{-1}\left(1 + \frac{y^2}{8}\right)\left(1 - 2\sqrt{p_1} e^{-p_1} \int_{i\infty}^{\sqrt{p_1}} e^{w^2} dw\right) \frac{e^{ikr}}{r} \tag{38}$$

where,

$$r > \lambda; \ |y^4| \ll 1.$$

Since y^2 does not vary with distance and is always very small, the constant terms $(1+y^2)^{-1}$ $(1+y^2/8)$ may be neglected for most practical purposes so that

$$|F| = \frac{F_1}{r}\left|1 - 2\sqrt{p_1} e^{-p_1} \int_{i\infty}^{\sqrt{p_1}} e^{w^2} dw\right| \tag{39}$$

where,

$$r > \lambda; \ |y^4| \ll 1$$

$$A = \left|1 - 2\sqrt{p_1} e^{-p_1} \int_{i\infty}^{\sqrt{p_1}} e^{w^2} dw\right| \tag{40}$$

where,

$$r > \lambda; \ |y^4| \ll 1.$$

Equation (40) is exactly the formula given by Sommerfeld in his 1926 paper[18] but is free from the error in sign which occurs in his 1909 paper.[1] This error was discussed by the author in a recent article.[4] However, Sommerfeld's formula was restricted to the case $|y^2| \ll 1$ while (40) is only restricted to the case $|y^4| \ll 1$ which permits its use for all physical applications even at the ultra-high frequencies. This restriction was removed by the use of a slightly different definition of the numerical distance.

With the same restriction $|y^4| \ll 1$ we have from (25)

$$p = \frac{\pi}{x} \frac{r}{\lambda} \cos b = \frac{\pi}{\epsilon + 1} \frac{r}{\lambda} \sin b \tag{41a}$$

$$\tan b = \frac{\epsilon + 1}{x}. \tag{42}$$

For low frequencies or good conductivities, when $x >> \epsilon + 1$, $b = 0$, $\cos b = 1$ and we have by (41a) the usual definition of the "numerical distance" indicating that the propagation is independent of the dielectric constant of the ground. In this case (40) may be written

$$A = \left| 1 - 2\sqrt{p}e^{-p} \int_0^{\sqrt{p}} e^{w^2} dw + i\sqrt{\pi p}e^{-p} \right| \tag{43}$$

where,

$$x \gg \epsilon + 1$$

$$p = \frac{\pi}{x} \frac{r}{\lambda}. \tag{41b}$$

The integral in (43) has been tabulated by H. G. Dawson.[19]

For high frequencies or poor conductivities when $x \ll \epsilon + 1$, $b = 90$ degrees, $\sin b = 1$ and we have by (41a)

$$p = \frac{\pi}{\epsilon + 1} \frac{r}{\lambda} \tag{41c}$$

where,

$$x \ll \epsilon + 1.$$

[18] *Ann. der Phys.*, vol. 81, p. 1135, (1926).
[19] *Proc. London Math. Soc.*, vol. 29, pp. 519–522, (1897–1898).

In this case (40) may be written

$$A = \left| 1 - \sqrt{\pi p}\, e^{-i\pi/4} e^{-ip} \{1 - c(p) - s(p) - i[s(p) - c(p)]\} \right| \qquad (44)$$

where,

$$x \ll \epsilon + 1$$

$$c(p) = \int_0^p \cos\left(\frac{\pi x^2}{2}\right) dx$$

$$s(p) = \int_0^p \sin\left(\frac{\pi x^2}{2}\right) dx$$

where $c(p)$ and $s(p)$ are Fresnel's integrals which are tabulated by Watson.[20]

For intermediate frequencies (40) was evaluated in terms of the error function of a complex variable.

$$A = \left| 1 + i\sqrt{\pi p_1}\, e^{-p_1} \operatorname{erfc}(-i\sqrt{p_1}) \right|. \qquad (45)$$

A short table of the error function for various complex values of the argument was furnished to the author by Ronald M. Foster of the American Telephone and Telegraph Company. A few additional points were computed by means of the series expansions of (40):

$$A = |u + iv| \qquad (46)$$

where,

$$u = 1 - 2p\cos b + \frac{(2p)^2}{1\cdot 3}\cos 2b - \frac{(2p)^3}{1\cdot 3\cdot 5}\cos 3b + \cdots$$
$$+ \sqrt{\pi p}\, e^{-p\cos b} \sin\left(p \sin b - \frac{b}{2}\right)$$

$$v = -2p\sin b + \frac{(2p)^2}{1\cdot 3}\sin 2b - \cdots \qquad (47)$$
$$+ \sqrt{\pi p}\, e^{-p\cos b} \cos\left(p \sin b - \frac{b}{2}\right).$$

The above equations are useful for short numerical distances (i.e., $p < 1$) while the following asymptotic expansions may be used for large numerical distances:

[20] G. N. Watson, "Bessel Functions," Cambridge, (1922), pp. 744–745.

$$
\begin{aligned}
u &= -\frac{\cos b}{2p} - \frac{1\cdot 3\cos 2b}{(2p)^2} - \frac{1\cdot 3\cdot 5\cos 3b}{(2p)^3} - \cdots \\
v &= \frac{\sin b}{2p} + \frac{1\cdot 3\sin 2b}{(2p)^2} + \cdots.
\end{aligned}
\tag{48}
$$

It should be emphasized that all of the above formulas refer to the field intensity at the surface of the earth and are based on the wave function for a short antenna. For antennas whose height is an appreciable fraction of a wave length long, the above theory breaks down at the nearer distances.

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MODIFIED SOMMERFELD'S INTEGRAL AND ITS APPLICATIONS*

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Summary—*The purpose of this paper is to obtain a certain integral expressing the fundamental wave function and with the aid of this integral to calculate the radiation resistances of small doublets and small loops placed inside an infinite hollow cylinder. Some applications of this integral to calculation of radiation from parallel wires in free space are also discussed.*

In 1909 Sommerfeld[1] applied the following integral

$$\frac{e^{-i\beta r}}{r} = \int_0^\infty e^{\pm z\sqrt{\xi^2-\beta^2}} J_0(\xi\rho) \frac{\xi d\xi}{\sqrt{\xi^2-\beta^2}}, \; z \gtrless 0, \; r^2 = \rho^2 + z^2,$$

to the problem of propagation of electric waves over an imperfect earth. The value of such a representation of the wave function corresponding to a simple point source in dealing with plane boundaries is well known.

This representation is not suitable for satisfying the boundary conditions in the case of cylindrical boundaries but there exists an analogous expression convenient for handling the problems involving such boundaries. Among the practical problems in the solution of which the modified Sommerfeld's integral proves its value are: (1) radiation of electromagnetic waves in a hollow cylindrical conductor, (2) radiation of waves from a source situated inside a medium consisting of coaxial cylindrical homogeneous layers of different substances, and (3) radiation from parallel wires in free space when the current distribution in such wires is known. In this paper we restrict ourselves to the first and the last problems only. The second problem admits of an easy formal solution; this solution is so complex, however, that its analysis and interpretation constitute a problem in their own right.

The following expression is the one to which we have referred as the modified Sommerfeld's integral

$$\frac{e^{-i\beta r}}{r} = \frac{2}{\pi} \int_{(C)} K_0(\rho\sqrt{\xi^2 - \beta^2}) \cos \xi z \, d\xi, \tag{1}$$

where,

$$r = \sqrt{\rho^2 + z^2} \tag{2}$$

* Decimal classification: R111.2 Original manuscript received by the Institute June 4, 1936.

[1] *Ann. der Phys.*, vol. 28, p. 683; March, (1909).

and the contour (C) is the positive real axis indented upward at $\xi=\beta$ (Fig. 1). The function K_0 is the modified Bessel function of the second kind.[2] We choose $0 \leqq ph(\xi^2-\beta^2) \leqq \pi$. The integral is convergent for all values of ρ different from zero.

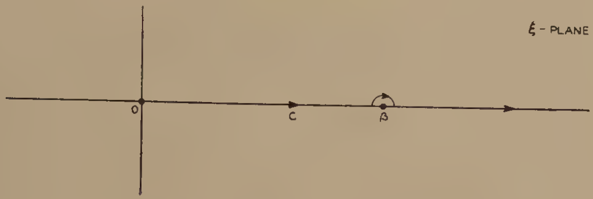

Fig. 1.—The contour of integration with an infinitely small semicircular indentation at $\xi=\beta$.

It is evident that both sides of (1) represent wave functions so that it will be sufficient to prove their identity for $z=0$. Thus let us calculate

$$A - jB = \frac{2}{\pi}\int_{(C)} K_0(\rho\sqrt{\xi^2-\beta^2})d\xi. \tag{3}$$

To the left of $\xi=\beta$ the integrand is $K_0(j\rho\sqrt{\beta^2-\xi^2})$. Since

$$\begin{aligned} K_0(j\rho\sqrt{\beta^2-\xi^2}) &= -\frac{j\pi}{2}H_0{}^2(\rho\sqrt{\beta^2-\xi^2}) = \\ -\frac{\pi}{2}N_0(\rho\sqrt{\beta^2-\xi^2}) &- \frac{j\pi}{2}J_0(\rho\sqrt{\beta^2-\xi^2}), \end{aligned} \tag{4}$$

we have[3]

$$\begin{aligned} A &= -\int_0^\beta N_0(\rho\sqrt{\beta^2-\xi^2})d\xi + \frac{2}{\pi}\int_\beta^\infty K_0(\rho\sqrt{\xi^2-\beta^2})d\xi \\ B &= \int_0^\beta J_0(\rho\sqrt{\beta^2-\xi^2})d\xi. \end{aligned} \tag{5}$$

The integrals in question are convergent and the contribution of the semicircle at $\xi=\beta$ vanishes with the radius of the indentation.

[2] The relation (1) can be obtained from No. 868 of Fourier Integrals for Practical Applications by G. A. Campbell and R. M. Foster or from equation (2) on page 416 of The Theory of Bessel Functions by G. N. Watson, The practical value of (1) in certain problems is so great, however, that a brief proof of it is here included.

[3] The Bessel function of the second kind is designated after Jahnke and Emde by $N_n(x)$.

A simple change of variables $\xi=\beta\cos\psi$, expansion of the integrand into a power series and integration, will lead at once to $B=\sin\beta\rho/\rho$.

The calculation of A is somewhat more complicated. Making use of the following expressions

$$N_0(x) = -\frac{2}{\pi}\int_x^\infty \frac{\cos t\,dt}{\sqrt{t^2-x^2}}\,dt, \qquad K_0(x) = \int_0^\infty \frac{\cos t\,dt}{\sqrt{t^2+x^2}} \tag{6}$$

and introducing new variables

$$t = R\cos\psi, \qquad \rho\xi = R\sin\psi, \tag{7}$$

we obtain

$$\begin{aligned} A &= \frac{2}{\pi\rho}\int_0^{\pi/2}\int_{\beta\rho}^\infty \frac{\cos(R\cos\psi)}{\sqrt{R^2-\beta^2\rho^2}}R\,dR\,d\psi = \frac{1}{\rho}\int_{\beta\rho}^\infty \frac{RJ_0(R)}{\sqrt{R^2-\beta^2\rho^2}}dR \\ &= \frac{1}{\rho}\int_0^\infty J_0(\sqrt{u^2+\beta^2\rho^2})du = \frac{\cos\beta\rho}{\rho}. \end{aligned} \tag{8}$$

The final step may be proved by employing the addition theorem for Bessel functions, integrating and identifying the resulting series. Having obtained the fundamental identity (1) we proceed to its applications.

Radiation Inside a Perfectly Conducting Cylinder

In this section we shall be concerned with radiation from electric and magnetic current elements, situated inside a perfectly conducting cylindrical surface of radius a. In what follows we shall assume that these elements are parallel to the axis of the cylinder. A magnetic current element is, of course, equivalent to an electric current loop.

First let us consider an electric current element of moment Il, at a distance b from the axis, I being the current and l the length of the element. The vector potential of this current element is

$$A_z = \Pi = \frac{Ile^{-i\beta r'}}{4\pi r'}, \qquad A_\rho = A_\phi = 0, \tag{9}$$

where r' is the distance AP between the element and P. The phase constant β is equal to 2π divided by the wave length λ. The longitudinal component of the electric intensity is

$$E_z = \frac{1}{j\omega\epsilon}\left(\beta^2\Pi + \frac{\partial^2\Pi}{\partial z^2}\right) \tag{10}$$

where ϵ is the dielectric constant.

From (1), with the aid of the addition theorem for modified Bessel functions, we have[4]

$$\Pi = \frac{Il}{2\pi^2} \sum_{n=0}^{\infty} \epsilon_n \cos n\phi \int_{(C)} I_n(b\sqrt{\xi^2 - \beta^2}) K_n(\rho\sqrt{\xi^2 - \beta^2}) \cos \xi z \, d\xi, \quad (11)$$

provided ρ is greater than b. From this we find

$$E_z = \frac{Il}{2\pi^2 j\omega\epsilon} \sum_{n=0}^{\infty} \epsilon_n \cos n\phi \int_{(C)} (\beta^2 - \xi^2) I_n(b\sqrt{\xi^2 - \beta^2}) K_n(\rho\sqrt{\xi^2 - \beta^2}) \cos \xi z \, d\xi. \quad (12)$$

The reflected field must be free from singularities for $\rho < a$. Hence the general form for the reflected longitudinal electric intensity is

$$E_z' = \sum_{n=0}^{\infty} \epsilon_n \cos n\phi \int_{(C)} P(\xi) I_n(\rho\sqrt{\xi^2 - \beta^2}) \cos \xi z \, d\xi, \quad (13)$$

where $P(\xi)$ is an unknown function. At the surface $\rho = a$ the total longitudinal component of E must vanish; therefore,

$$E_z' = \frac{Il}{2\pi^2 j\omega\epsilon} \sum_{n=0}^{\infty} \epsilon_n \cos n\phi \int_{(C)} \frac{(\xi^2 - \beta^2) I_n(b\sqrt{\xi^2 - \beta^2}) K_n(a\sqrt{\xi^2 - \beta^2}) I_n(\rho\sqrt{\xi^2 - \beta^2})}{I_n(a\sqrt{\xi^2 - \beta^2})} \cos \xi z \, d\xi. \quad (14)$$

The mutual impedance between the cylinder and the current element is the electromotive force which must be applied to the element in order to counteract the reaction of the cylinder, divided by the current I; this mutual impedance Z_M is

$$Z_M = -\frac{lE_z'(b, 0, 0)}{I} \text{ ohms.} \quad (15)$$

From (14) we obtain the following expression for the mutual impedance

$$Z_M = \frac{l^2}{2\pi^2 j\omega\epsilon} \sum_{n=0}^{\infty} \epsilon_n \int_{(C)} (\beta^2 - \xi^2) \frac{I_n^2(b\sqrt{\xi^2 - \beta^2}) K_n(a\sqrt{\xi^2 - \beta^2})}{I_n(a\sqrt{\xi^2 - \beta^2})} \, d\xi. \quad (16)$$

The real part of Z_M is the mutual radiation resistance of the current element and the cylinder.

[4] The Neumann number $\epsilon_n = 2$ for any n different from zero; $\epsilon_0 = 1$. By means of the addition theorem we obtain expressions that make it easy for us to satisfy the boundary conditions at $\rho = a$.

In order to interpret Z_M we shall separate its real and imaginary parts. The integrand is evidently real on the right of $\xi=\beta$. On the left of this point we have

$$\begin{aligned} I_n(b\sqrt{\xi^2-\beta^2}) &= I_n(jb\sqrt{\beta^2-\xi^2}) = j^n J_n(b\sqrt{\beta^2-\xi^2}), \\ K_n(a\sqrt{\xi^2-\beta^2}) &= K_n(ja\sqrt{\beta^2-\xi^2}) \\ &= -\frac{\pi}{2}(-j)^n[N_n(a\sqrt{\beta^2-\xi^2}) + jJ_n(a\sqrt{\beta^2-\xi^2})]. \end{aligned} \tag{17}$$

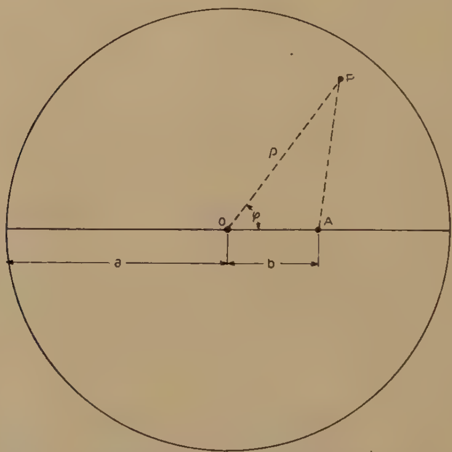

Fig. 2—The cross section of a perfectly conducting cylindrical surface. The z-axis is toward the reader.

Introducing these expressions into (16) and remembering that the contribution of an infinitely small indentation at $\xi=\beta$ is nil, we obtain

$$\begin{aligned} Z_M = {} & \frac{l^2}{4\pi j\omega\epsilon}\sum_{n=0}^{\infty}\epsilon_n\int_0^{\beta}(\xi^2-\beta^2)\frac{J_n^2(b\sqrt{\beta^2-\xi^2})N_n(a\sqrt{\beta^2-\xi^2})}{J_n(a\sqrt{\beta^2-\xi^2})}d\xi \\ & + \frac{l^2}{4\pi\omega\epsilon}\sum_{n=0}^{\infty}\epsilon_n\int_0^{\beta}(\xi^2-\beta^2)J_n^2(b\sqrt{\beta^2-\xi^2})d\xi \\ & + \frac{l^2}{2\pi^2 j\omega\epsilon}\sum_{n=0}^{\infty}\epsilon_n\int_{\beta}^{\infty}(\beta^2-\xi^2)\frac{I_n^2(b\sqrt{\xi^2-\beta^2})K_n(a\sqrt{\xi^2-\beta^2})}{I_n(a\sqrt{\xi^2-\beta^2})}d\xi. \end{aligned} \tag{18}$$

Since the integrand of the first term may have poles on the real axis, the contour of integration must be indented at these points. On account of our previous agreement concerning the phase of $\xi^2-\beta^2$, these indentations must be upward; we may assume them to be infinitely small semicircles.

The second term can be evaluated at once; thus

$$\sum_{n=0}^{\infty} \epsilon_n \int_0^{\beta} (\xi^2 - \beta^2) J_n^2(b\sqrt{\beta^2 - \xi^2}) d\xi = \int_0^{\beta} (\xi^2 - \beta^2) d\xi = -\frac{2}{3}\beta^3. \quad (19)$$

Consequently, the middle term is equal to $-(2/3)\pi\eta\ (l/\lambda)^2$ where $\eta = \sqrt{\mu/\epsilon}$ is the intrinsic impedance of the medium in ohms. This term is exactly the negative of the radiation resistance of the current element in the absence of the cylindrical envelope. Thus the total impedance of the current element will consist of the first and the last terms of Z_M and of the self-reactance of the element. The last term in (18) is always a pure imaginary. The first term is also a pure imaginary so long as the frequency is such that βa is less than the smallest root of $J_0(x) = 0$. Thus the law of radiation inside a perfectly conducting cylinder is quite different from that in an unlimited medium. There is *no radiation whatsoever* for frequencies below a certain critical frequency f_c determined by the equation

$$\beta_c a = \frac{2\pi f_c a}{c} = 2 \cdot 40 \cdots \quad (20)$$

in which c is the velocity of light appropriate to the medium. This frequency and the corresponding wave length are

$$f_c = \frac{2.40c}{2\pi a}, \qquad \lambda_c = \frac{2\pi a}{2.40 \cdots}. \quad (21)$$

But when the frequency is high enough to make βa exceed the smallest zero of $J_0(x)$, a real term will be contributed by the first integral in Z_M. It comes from the integration around the semicircular indentation over the pole of the integrand. As the frequency increases, more and more poles of the integrand fall within the range $(0, \beta a)$, each contributing a real term. The reactive part is simply Cauchy's principal value of the integral.

For actual calculation of the radiation resistance it is convenient to introduce a new variable $t = a\sqrt{\beta^2 - \xi^2}$ into the first term of Z_M; thus we obtain

$$Z_M' = -\frac{l^2}{4\pi i\omega\epsilon a^3} \sum_{n=0}^{\infty} \epsilon_n \int_0^{\beta a} \frac{t^3 J_n^2\left(\frac{b}{a} t\right) N_n(t)}{\sqrt{\beta^2 a^2 - t^2}\, J_n(t)} dt. \quad (22)$$

Let $k_{n,m}$ be the mth zero of $J_n(x)$ and let $k_{n,m} < \beta a$. The contribution $R_{n,m}$ to the real part of Z_M coming from the integration over the pole $\xi = k_{n,m}/a$ is evidently

$$R_{n,m} = -\frac{\epsilon_n k_{n,m}{}^3 J_n{}^2\left(\dfrac{b}{a}k_{n,m}\right) N_n(k_{n,m}) l^2}{4\omega\epsilon a^3 J_n{}'(k_{n,m})\sqrt{(\beta a)^2 - k_{n,m}{}^2}} \text{ ohms.} \tag{23}$$

The frequency defined by the equation

$$\frac{\omega_{n,m} a}{c} = k_{n,m}, \qquad \omega_{n,m} = \frac{c k_{n,m}}{a}, \qquad \lambda_{n,m} = \frac{2\pi a}{k_{n,m}}, \tag{24}$$

is one of the critical frequencies at which a resonance will occur. Designating by $\nu_{n,m}$ the ratio of this critical frequency and the applied frequency and utilizing the identity

$$N_n(k_{n,m}) = -\frac{2}{\pi k_{n,m} J_n{}'(k_{n,m})} \tag{25}$$

we can transform (23) into

$$R_{n,m} = \eta \frac{\epsilon_n}{2\pi}\left[\frac{J_n\left(k_{n,m}\dfrac{b}{a}\right)}{J_n{}'(k_{n,m})}\right]^2 \left(\frac{l}{a}\right)^2 \frac{\nu_{n,m}{}^2}{\sqrt{1-\nu_{n,m}{}^2}} \text{ ohms.} \tag{26}$$

The total radiation resistance of the electric current element is the sum of all $R_{n,m}$ for which $k_{n,m}$ is less than βa.

The radiation from a magnetic current element can be investigated along parallel lines. Here we start with an electric vector potential

$$F_z = \Pi = \frac{Kle^{-j\beta r'}}{4\pi r'} = \frac{j\omega\mu SIe^{-j\beta r'}}{4\pi r'}, \quad F_\rho = F_\phi = 0, \tag{27}$$

in which Kl is the moment of the element, K being the magnetic current and l the length of the element. Such an element is equivalent to an electric current loop of area S, carrying current I amperes, provided $Kl = j\omega\mu SI$. The longitudinal component of the magnetomotive intensity is then

$$H_z = \frac{1}{j\omega\mu}\left(\frac{\partial^2\Pi}{\partial z^2} + \beta^2\Pi\right). \tag{28}$$

At the boundary of the cylinder the normal component of the total magnetomotive intensity must vanish. This is equivalent to saying that $\partial/\partial\rho\ (\Pi+\Pi')$, where Π' is the reflected vector potential, must vanish there. In order to find the proper function Π' we express Π in

the form (11), postulate the proper form for Π', and find the unknown function in the integrand for Π' so that the boundary condition is fulfilled. In this way we find the longitudinal magnetomotive intensity of the reflected field

$$H_z'(\rho, \phi, z) = \frac{SI}{2\pi^2} \sum_{n=0}^{\infty} \epsilon_n \cos n\phi$$

$$\int_{(C)} (\xi^2 - \beta^2) \frac{I_n(b\sqrt{\xi^2 - \beta^2}) K_n'(a\sqrt{\xi^2 - \beta^2}) I_n(\rho\sqrt{\xi^2 - \beta^2})}{I_n'(a\sqrt{\xi^2 - \beta^2})} \cos \xi z \, d\xi. \tag{29}$$

The electromotive force which must be applied to the loop in order to counteract that induced by the electric currents flowing in the cylinder is equal to $j\omega\mu S\, H_z'(b, 0, 0)$; this electromotive force divided by the current in the loop is the mutual impedance Z_M between the loop and the cylinder. Hence we have

$$Z_M = \frac{j\omega\mu S^2}{2\pi^2} \sum_{n=0}^{\infty} \epsilon_n \int_{(C)} (\xi^2 - \beta^2) \frac{I_n^2(b\sqrt{\xi^2 - \beta^2}) K_n'(a\sqrt{\xi^2 - \beta^2})}{I_n'(a\sqrt{\xi^2 - \beta^2})} d\xi. \tag{30}$$

The real part of this impedance is the mutual radiation resistance. Transforming (30), we obtain

$$\begin{aligned} Z_M = &- \frac{j\omega\mu S^2}{4\pi} \sum_{n=0}^{\infty} \epsilon_n \int_0^{\beta} (\xi^2 - \beta^2) \frac{J_n^2(b\sqrt{\beta^2 - \xi^2}) N_n'(a\sqrt{\beta^2 - \xi^2})}{J_n'(a\sqrt{\beta^2 - \xi^2})} d\xi \\ &- \frac{8\pi^3}{3} \eta \left(\frac{S}{\lambda^2}\right)^2 \\ &+ \frac{j\omega\mu S^2}{2\pi^2} \sum_{n=0}^{\infty} \epsilon_n \int_{\beta}^{\infty} (\xi^2 - \beta^2) \frac{I_n^2(b\sqrt{\xi^2 - \beta^2}) K_n'(a\sqrt{\xi^2 - \beta^2})}{I_n'(a\sqrt{\xi^2 - \beta^2})} d\xi. \end{aligned} \tag{31}$$

The second term is equal and opposite to the radiation resistance of the loop in the absence of the cylinder; the last term is always a pure reactance; but the first term may contribute to the total radiation resistance of the loop. The first term is reactive so long as the frequency is such that βa is less than the smallest root of $J_n'(x) = 0$ since in this case there are no poles in the path of integration. The smallest root of this set of equations occurs when $n = 1$; this root is approximately equal to 1.84. As in the preceding investigation we find that the total radiation resistance of an electric current loop with its axis parallel to the axis of the surrounding cylinder is equal to the sum of contributions $R_{n,m}$ coming from the integration over those poles $\xi = k_{n,m}/a$, where $J_n'(k_{n,m}) = 0$, which lie in the range $(0, \beta)$. These contributions are

$$R_{n,m} = \frac{\epsilon_n}{2\pi}\eta\frac{k_{n,m}^4}{k_{n,m}^2 - n^2}\left[\frac{J_n\left(\frac{b}{a}k_{n,m}\right)}{J_n(k_{n,m})}\right]^2\left(\frac{S}{a^2}\right)^2(1 - \nu_{n,m}^2)^{-1/2} \text{ ohms.} \tag{32}$$

It is not easy to calculate from (31) the reactive part of the mutual radiation impedance. Besides, the total reactance is materially affected by the self-reactance of the element, depending upon the radius of the element and the capacity of its terminal condensers. Fortunately it is usual to tune the radiating circuits and calculation of the reactance is of lesser importance than calculation of the radiation resistance.

The reactive part of the radiation impedance of a circuit is, of course, nothing else but the correct value of the circuit's external reactance—the external as distinguished from the internal, the latter being due to the flux interlinkages within the wire itself. In fact, the "radiation impedance" is the name under which we know the correct value of the external impedance of a circuit. When the circuit is small compared with the wave length, the radiation impedance is substantially equal to the reactance calculated with the aid of the well-known Neumann's formula.

Radiation from Parallel Wires

Whenever the current distribution in a system of parallel wires is given, it is a simple matter to calculate the field and the radiation resistance with the aid of the fundamental integral (1). It has been found that the electric current in such wires is substantially sinusoidal so that we shall restrict ourselves to this case.

To begin with we shall consider a wire of radius a and length $2l$ and assume that the generator is at the center of the wire. Thus the current distribution will be symmetrical on both sides of the generator and in the upper half of the wire we have

$$I(z') = I \sin \beta(l - z'), \tag{33}$$

where z' is the distance from the generator.

Supposing that the current density is independent of ϕ and utilizing (12), we obtain after routine calculations the longitudinal electric intensity

$$E_z = \frac{j\eta I}{2\pi^2}\int_{(C)} [2 \cos \beta l \cos \xi z - \cos \xi(l - z) \\ - \cos \xi(l + z)]I_0(a\sqrt{\xi^2 - \beta^2})K_0(\rho\sqrt{\xi^2 - \beta^2})d\xi. \tag{34}$$

When the wire is infinitely thin, we let $a = 0$. Comparing the resulting expression with (1), we obtain

$$E_z = \frac{j\eta I}{2\pi}\left(\frac{e^{-j\beta r}}{r}\cos\beta l - \frac{e^{-j\beta r_1}}{2r_1} - \frac{e^{-j\beta r_2}}{2r_2}\right); \tag{35}$$

where r, r_1, and r_2 are respectively the distances of a typical point in space from the center of the wire and from its ends.

Expression (35) can be generalized for a wire of finite radius. We merely note that

$$\left(\frac{\partial^2}{\partial z^2} + \beta^2\right)^n \frac{e^{-j\beta r}}{r} = \frac{2}{\pi}\int_{(C)} (\beta^2 - \xi^2)^n K_0(\rho\sqrt{\xi^2 - \beta^2}) \cos \xi z \, d\xi \tag{36}$$

and that

$$I_0(a\sqrt{\xi^2 - \beta^2}) = \sum_{n=0}^{\infty} \frac{(-)^n(\beta^2 - \xi^2)^n a^{2n}}{2^{2n}(n!)^2}. \tag{37}$$

Substituting (37) into (34) and making use of (36) we have

$$E_z = \frac{j\eta I}{2\pi}\sum_{n=0}^{\infty}\frac{(-)^n a^{2n}}{2^{2n}(n!)^2}\left(\frac{\partial^2}{\partial z^2} + \beta^2\right)^n\left(\frac{e^{-j\beta r}}{r}\cos\beta l - \frac{e^{-j\beta r_1}}{2r_1} - \frac{e^{-j\beta r_2}}{2r_2}\right). \tag{38}$$

The distances r, r_1, and r_2 are counted from the center and the ends of the axis of the wire.

In order to obtain an expression for the radiated power we need only integrate the product of the electric intensity and the conjugate of the current density over the surface of the wires. One half of the real part of such an integral will represent the average radiated power. The general equation for the radiated power can be conveniently expressed in the form

$$W = \sum_{m,k}^{n}\sum W_{mk}, \tag{39}$$

where W_{mk} is the power radiated by the kth wire by virtue of its interaction with the field produced by the mth wire.

If a_m and a_k are the radii of two parallel wires and if $s_{m,k}$ is their interaxial separation, then the mutual radiated power is

$$W_{mk} = \frac{1}{4\pi\omega\epsilon}\int_0^\beta (\beta^2 - \xi^2)P(\xi)J_0(a_m\sqrt{\beta^2 - \xi^2}) \\ J_0(s_{mk}\sqrt{\beta^2 - \xi^2})J_0(a_k\sqrt{\beta^2 - \xi^2})d\xi. \tag{40}$$

The function $P(\xi)$ depends on the current distribution along the wires; thus[5]

[5] The asterisk designates the conjugate complex of I.

$$P(\xi) = Re \int_{l_m'}^{l_m''} \int_{l_k'}^{l_k''} I(z_m)I^*(z_k) \cos \xi(z_m - z_k)dz_m\, dz_k, \tag{41}$$

where the l's represent the co-ordinates of the ends of the axes of the wires. Since the derivation of (40) is simple and straightforward we shall omit it.

Conclusion

The integral (1) permits a formal solution for Green's function in a medium comprised of homogeneous coaxial cylindrical layers. As it stands it applies only to nondissipative media. The general formula, applicable to dissipative media, is

$$\frac{e^{-\sigma r}}{r} = \frac{2}{\pi} \int_{(C)} K_0(\rho\sqrt{\xi^2 + \sigma^2}) \cos \xi z\, d\xi \tag{42}$$

where σ is the intrinsic propagation constant

$$\sigma = \sqrt{j\omega\mu(g + j\omega\epsilon)},$$

g being the conductivity of the medium. The interpretation of the results becomes quite difficult, however.

BOOKLETS, CATALOGS, AND PAMPHLETS RECEIVED

Copies of the publications listed on this page may be obtained without charge by addressing the publishers.

The first issue of "HK Pay Dirt" gives technical data on a new type 154 gammatron tube and is available from Heintz and Kaufman of South San Francisco, Calif.

Hygrade Sylvania Corporation of Emporium, Pa., has issued Engineering News Letter No. 26 on degeneration of audio amplifiers, No. 27 on output characteristics of Type 1E7G, No. 28 on the correlation of tube types providing "equivalent" services, and No. 29 on improving the performance of battery-operated receivers. Technical data sheets have been issued on the 6B8 and 6B8G, duodiode high gain pentode; 6L6G, power amplifier; and 25B6g, power amplifier pentode.

National Union Radio Corporation of 365 Ogden Street, Newark, N. J., has issued Engineering Bulletins on the 6D8G, self-excited electron-coupled converter; 6L5G, medium-mu voltage amplifier; 6N5, tuning indicator, 6Q6G, diode high-mu triode; and 6S7G, remote cutoff amplifier.

Raytheon Production Corporation of 55 Chapel Street, Newton, Mass., has issued data sheets on the following: 1D5G, variable-mu pentode amplifier; 1F4, pentode output tube; 1F5, pentode output tube; 6B8, all-metal duodiode pentode; 6D8G, pentagrid converter; 6L5G, detector amplifier triode; 6L6G, tetrode power amplifier; 6N5, electron ray tube; 6Q6G, single diode high-mu triode; 6S7G, triple grid supercontrol amplifier; 950, pentode output tube; RK-33, dual triode; RK-36, transmitting tube.

The RCA Manufacturing Company of Harrison, N. J., has issued Application Notes No. 60 on the operation of the 6L6; No. 61 on the conversion of the 6L6 plate family to new screen-grid voltage conditions; and No. 62 on the new operating condition for two type 6F6 tubes connected as pentodes; No. 63 on the high gain single tube phase converter; No. 64 on inverse feed-back circuits for audio-frequency amplifiers. Data sheets have been issued on the 1F4 power amplifier pentode; 1F6, duplex diode pentode; 5W4, full-wave rectifier; and 6N7, class B twin amplifier.

Leaflets have been issued by Shure Brothers of 215 W. Huron Street, Chicago, Ill., on their four-way utility microphone.

A leaflet has been issued by The Triplett Electrical Instrument Company of Bluffton, Ohio on their new master units which include a volt-ohm-milliammeter, tube tester, free-point tester, and direct reading all-wave signal generator.

Variac transformers are described in a leaflet issued by the General Radio Company of 30 State Street, Cambridge, Mass.

Aircraft radio compass Model AVR-8, general purpose airport receiver Model AVR-11, and an AVA-9 conversion kit for modifying an AVR-7 or AVR7-A aircraft receiver are described in leaflets issued by the Aviation Radio Section of the RCA Manufacturing Company of Camden, N. J.

The Type NC-100 superheterodyne for the frequency range of 540 kilocycles to 30 megacycles is described in a leaflet issued by the National Company of Malden, Mass.

A booklet on Lamicoid electrical insulating materials has been issued by the Mica Insulator Company of 200 Varick Street, New York, N. Y.

Information Bulletins No. 4 on high-frequency tubes and No. 5 on the sterilamp have been issued by Westinghouse Lamp Company of Bloomfield, N. J.

The Roller-Smith Company of 233 Broadway, New York, N. Y., has issued Catalog No. 10 covering three types of voltage regulators. Air circuit breakers are listed in Catalog 5. Catalog 123 covers a complete line of portable measuring instruments and Catalog 830 covers recording instruments.

A complete line of servicing equipment is described in some leaflets issued by the Supreme Instruments Corporation of Greenwood, Miss.

Dynamic microphones are covered in a leaflet issued by the Radio Receptor Company of 106 Seventh Avenue, New York, N. Y.

The Webster Company of 3825 West Lake Street, Chicago, Ill., has issued a catalog on their sound systems and equipment.

A leaflet describes the BR2S sound cell microphone produced by the Brush Development Company of 1899 E. 40 Street, Cleveland, Ohio.

"Western Electric Vacuum Tubes for use in Amateur Radio Telephone Transmitting Equipment" is the name of a booklet issued by the Western Electric Company of 195 Broadway, New York, N. Y.

Insuline Corporation of America of 25 Park Place, New York, N. Y., has issued Catalog No. 190 covering its line of radio equipment and parts.

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